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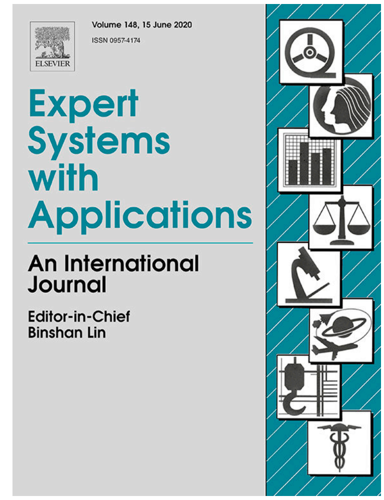

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An Improved Grey Wolf Optimizer for Solving Engineering Problems

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Abstract: In this article, an Improved Grey Wolf Optimizer (I-GWO) is proposed for solving global optimization and engineering design problems. This improvement is proposed to alleviate the lack of population diversity, the imbalance between the exploitation and exploration, and premature convergence of the GWO algorithm. The I-GWO algorithm benefits from a new movement strategy named dimension learning-based hunting (DLH) search strategy inherited from the individual hunting behavior of wolves in nature. DLH uses a different approach to construct a neighborhood for each wolf in which the neighboring information can be shared between wolves. This dimension learning used in the DLH search strategy can enhance the balance between local and global search and maintains diversity. The performance of the proposed I-GWO algorithm is evaluated on the CEC 2018 benchmark suite and four engineering problems. In all experiments, I-GWO is compared with six other state-of-the-art metaheuristics. The results are also analyzed by Friedman and MAE statistical tests. The experimental results and statistical tests demonstrate that the I-GWO algorithm is very competitive and often superior compared to the algorithms used in the experiments. The results of the proposed algorithm on the engineering design problems demonstrate its efficiency and applicability.

Keywords: *Optimization, Metaheuristic, Swarm intelligence algorithm, Grey wolf optimizer, Improved grey wolf optimizer, Engineering optimization problems.*

1. Introduction

In the area of optimization, solving an optimization problem typically means finding optimal values for the decision variables to maximize or minimize a set of objective functions while not violating constraints. Most of the real-world optimization problems have several difficulties, including but not limited to, high computational cost, non-linear constraints, non-convex search landscape, dynamic/noisy objective functions, and large solution space (Fister Jr et al., 2013). These challenges are the main criteria to choose either exact or approximate algorithms for solving complex problems. Although the exact algorithms are capable of precisely providing the global optimum, their execution time is exponentially increased proportional to the number of variables (Talbi, 2009). On the contrary, the stochastic optimization algorithms are able to identify optimum or near-optimum solutions within a reasonable time. Heuristic and metaheuristic algorithms are recognized as one of the most practical branches of approximate algorithms, which are capable of solving complex problems (Sörensen, 2015). The metaheuristic algorithms can be classified into two categories non-nature-inspired and nature-inspired algorithms. Although a few algorithms have been developed in the first category such as the tabu search (TS) (Glover, 1989, 1990), iterated local search (ILS) (Lourenço et al., 2003), and adaptive dimensional search (ADS) (Hasançebi et al., 2015), many metaheuristic algorithms have been inspired by nature.

A wide variety of the nature-inspired algorithms have been introduced such as differential evolution (DE) (Storn et al., 1997), particle swarm optimization (PSO) (Eberhart et al., 1995), artificial bee colony (ABC) (Karaboga et al., 2007), krill herd (KH) (Gandomi et al., 2012), and gravitational search algorithm (GSA) (Rashedi et al., 2009). They are flexible and simple by nature for solving complex problems with continuous search space. Also, some metaheuristics such as genetic algorithm (GA) (Holland, 1992) and ant colony optimization (ACO) (Dorigo et al., 2008), were proposed for binary and combinatorial optimization. Moreover, different methods were employed to develop the binary version of a continuous algorithm (Taghian et al., 2018). The metaheuristic algorithms are applied for solving complex problems in different applications such as optimal power flow (Attia et al., 2018; A.-A. A. Mohamed et al., 2017; Nuaekaew et al., 2017), chip design (Fard et al., 2014; Venkataraman et al., 2020), feature selection (Arora et al., 2019; Faris, Mafarja, et al., 2018; Mafarja et al., 2019; Mafarja et al., 2018; Taghian et al., 2019a, 2019b; Taradeh et al., 2019), diseases diagnosis (Arjenaki et al., 2015; Gunasundari et al., 2016; Muthukaruppan et al., 2012; Shen et al., 2016; Zamani et al., 2016b), tour planning (Banaie-Dezfouli et al., 2018) and engineering optimization (He et al., 2020; He et al., 2019; Li et al., 2020; Wu et al., 2019).

The grey wolf optimizer (GWO) (Mirjalili et al., 2014) is a successful nature-inspired metaheuristic, which was recently proposed based on the leadership hierarchy and group hunting mechanism of the grey wolves in nature. The GWO has been regarded as an effective metaheuristic, and it has been applied in solving different optimization problems in many fields such as engineering, machining learning, medical, and bioinformatics (Faris, Aljarah, et al., 2018). In GWO, the search process is guided by three best wolves in each iteration, which shows a strong convergence toward these wolves. In contrast, it suffers from the lack of the population diversity, imbalance between the exploitation and exploration, and the premature convergence (Heidari et al., 2017; Lu et al., 2018; Tu et al., 2019a).

To overcome these weaknesses, in this paper, an enhancement of GWO named improved grey wolf optimizer (I-GWO) is proposed. The I-GWO improves the hunting search strategy of wolves by using a new search strategy named dimension learning-based hunting (DLH). The DLH search strategy is inspired by the individual hunting behavior of wolves in nature, and it increases the domain of global search by multi neighbors learning. Then, in each iteration, the I-GWO has both candidate wolves generated by the DLH and the GWO search strategies to move the wolf X_i from the current position to a better position. In addition, the I-GWO uses an additional selecting and updating step to select the winner candidate wolf in each iteration and update the current position for the next iteration.

In the rest of the paper, the related works are discussed and criticized in Section 2. Section 3 briefly presents the mathematical models of the GWO algorithm. The proposed I-GWO algorithm is proposed in Section 4. Section 5 thoroughly presents and analyses the experimental results on benchmark functions, and the significance of the results is proved by statistical analysis, respectively. The applicability of the I-GWO for solving real application problems is tested by engineering problems in Section 6. Section 7 discusses the main reasons for successes of the I-GWO and the DLH search strategy. Finally, the conclusions and future works are given in Section 8.

2. Related work

There are different ways to classify and describe metaheuristic algorithms. One way is to differentiate between the origin of them, which can be classified into two categories non-nature-inspired and nature-inspired algorithms. The non-nature-inspired algorithms are mostly based on an individual idea neither based on nor associated with any natural or social phenomena. There have been proposed a few successful algorithms in this category, such as the tabu search (TS) (Glover, 1989, 1990), iterated local search (ILS) (Lourenço et al., 2003), and adaptive dimensional search (ADS) (Hasançebi et al., 2015). The TS algorithm applies a traditional local search strategy enhanced by memory structures to store information about solutions visited in the past. It can promote the diversification when it does not allow returning to the recently visited solutions. The ILS algorithm is an improved hill-climbing algorithm to decrease the probability of getting stuck in the local optima; however, it can be trapped by multimodal problems. In ADS, to control the algorithm's convergence rate during the optimization process, the search dimensionality ratio is adaptively updated. It attempts to balance between the exploration and exploitation characteristics of the ADS during its search in the design space based on the technique's performance at each iteration. These non-nature-inspired algorithms do not guarantee to find the optimum solution, and they probably trap in the local optimum, they strive to avoid the generation of inadequate quality solutions. To keep focusing on the main objective of this study, the rest of this section is to review the popular related works in the category of nature-inspired algorithms.

Mother Nature, is the most significant problem solver, and it is the essential inspiring source to develop successful nature-inspired algorithms, which have been widely used for solving optimization problems (Zang et al., 2010). As shown in Fig. 1, in the literature (Del Ser et al., 2019; Mirjalili et al., 2016), these algorithms are classified into three categories: evolutionary, physics-based, and swarm intelligence algorithms. Evolutionary algorithms (EAs) represent a class of iterative optimization algorithms that simulate the evolution processes in nature (Talbi, 2009). The best individuals are combined to form a new generation, which is the main strength of EAs as it promotes the improvement of the population over the course of iterations. The most popular evolutionary algorithms are genetic algorithm (GA) (Holland, 1992) that simulates the Darwinian evolution, differential evolution (DE) (Storn et al., 1997), genetic programming (GP) (Koza, 1997), and evolution strategy (ES) (Rechenberg, 1973). Among them, the DE algorithm and its variants (Meng et al., 2019; Meng et al., 2018; Ali W Mohamed et al., 2019; Ali Wagdy Mohamed, 2015) have emerged as one of the most competitive families of the EAs.

Physics-based algorithms mimic physical rules in nature in which the individuals communicate around the search space by using concepts and laws of physics such as gravitational force, inertia force, light refraction law, and molecular dynamics. Some popular algorithms in this category are big bang-big crunch (BB-BC) (Erol et al., 2006), gravitational search algorithm (GSA) (Rashedi et al., 2009), charged system search (CSS) (Kaveh et al., 2010), ray optimization (RO) (Kaveh et al., 2012), black hole (BH) (Hatamlou, 2013), atom search optimization (ASO) (Zhao et al., 2019), and henry gas solubility optimization (Hashim et al., 2019).

Swarm intelligence algorithms (SIs) are inspired by the collective behavior of social creatures such as bird flocking, animal herding, and ants' foraging. All individuals with cooperation and interaction, collectively move toward the promising areas in the search space. Some of the most well-known algorithms in this category are particle swarm intelligence (PSO) (Eberhart et al., 1995), artificial bee colony (ABC) (Karaboga et al., 2007), krill herd (KH) (Gandomi et al., 2012), grey wolf optimizer (GWO) (Mirjalili et al., 2014), whale optimization algorithm (WOA) (Mirjalili et al., 2016), crow search algorithm (CSA) (Askarzadeh, 2016), and harris hawks optimization (HHO) (Heidari et al., 2019). These algorithms have been widely used to solve continuous or discrete optimization problems (Chen et al., 2018; Faris et al., 2015; Thaher et al., 2020; Zamani et al., 2016a).

Although SI algorithms have been proven effective while solving optimization problems, they may suffer from trapping in a local optimum, premature convergence, and loss of diversity in the solutions. Therefore, there have been proposed modified variations of SIs to tackle their weaknesses. The comprehensive learning particle swarm optimizer (CLPSO) (Liang et al., 2006) was proposed to exit from local optima, and the DEWCO (Elaziz et al., 2019) uses a hyper-heuristic for improving the initial population of WOA to increase its convergence speed. Also, the conscious neighborhood-based crow search algorithm (CCSA) (Zamani et al., 2019) strikes a balance between local and global search.

The grey wolf optimizer (GWO) was proposed in 2014 (Mirjalili et al., 2014), which is a population-based swarm intelligence algorithm that mimics the social hierarchy and the group hunting behavior of wolves. Due to its simplicity, employing fewer control parameters, and ease of implementation, GWO has been widely applied to solve different optimization problems such as parameters estimation (Mirjalili, 2015; X. Song et al., 2015), economic dispatch (Jayabarathi et al., 2016; Pradhan et al., 2016), unit commitment (Kamboj, 2016; Panwar et al., 2018), pattern recognition (Katarya et al., 2018), feature selection (Emary et al., 2015; Tu et al., 2019b), wind speed forecasting (J. Song et al., 2018), and optimal power flow (El-Fergany et al., 2015; Sulaiman et al., 2015).

Since the introduction of GWO in 2014, as shown in Fig. 1, a number of variants of the basic GWO algorithm have been proposed to overcome GWO's deficiencies and provide better performance. Saremi et al. (Saremi et al., 2015), proposed GWO-EPD by integrating the evolutionary population dynamics (EPD) operator and the canonical GWO algorithm. The EPD is used to remove the worst solutions and reposition them around the three best solutions of the population. However, it has premature convergence and loss of diversity, especially on hybrid and composition problems. Malik et al. (Malik et al., 2015), proposed the wdGWO algorithm, in which a weighted average of best solutions is computed instead of a simple arithmetic average. Jayabarathi et al. (Jayabarathi et al., 2016) introduced HGWO to solve the economic dispatch problem by using the mutation and crossover operators in GWO. It shows good performance to solve the constrained nonconvex problem, although it does not strike a proper balance between exploration and exploitation for solving composition functions. Saxena et al. presented E-GWO (Saxena et al., 2020), which uses a sinusoidal bridging mechanism in conjunction with tournament selection, crossover, and mutation. It shows a better exploration ability for landscapes with many local optima, but weak exploitation in unimodal problems and imbalanced exploration and exploitation in hybrid functions are still its major problems.

A number of GWO variants have been developed to avoid the local optima and accelerate convergence speed by modifying the mechanism of GWO. Mittal et al. (Mittal et al., 2016) proposed mGWO based on a nonlinear control parameter strategy, which focuses on a proper balance between exploration and exploitation. The movement strategy of mGWO is inherited from the GWO algorithm, so the algorithm might suggest from entrapment in locally optimal solutions and premature convergence. In another study, QOGWO (Guha et al., 2016), the quasi-oppositional based learning (Q-OBL) theory was integrated into the conventional GWO. Long et al. (Long et al., 2018) proposed EEGWO, in which the position updating mechanism of this algorithm was modified once again to alleviate its drawbacks. However, trapping in the local optima and premature convergence are still its major problems. Long et al. (Long et al., 2019) proposed ROL-GWO by a modified parameter C, which increased the exploration of the algorithm.

Singh et al. (Singh et al., 2017) proposed HGWOSCA, which benefit from the hybridization of GWO and SCA. The global convergence and also exploitation ability for the unimodal problems were improved, but its weak ability in the exploration for multimodal functions and also the balance for the composition functions remain. In another study, Gaidhane et al. (Gaidhane et al., 2018) proposed a GWO-ABC that uses the advantages of GWO and artificial bee colony (ABC). In this algorithm, the information sharing property of ABC is hybridized by the original hunting strategy of GWO to improve the exploration ability. Alomoush et al. (Alomoush et al., 2019) proposed a hybrid harmony search and GWO named GWO-HS with an opposition learning strategy to solve global optimization problems.

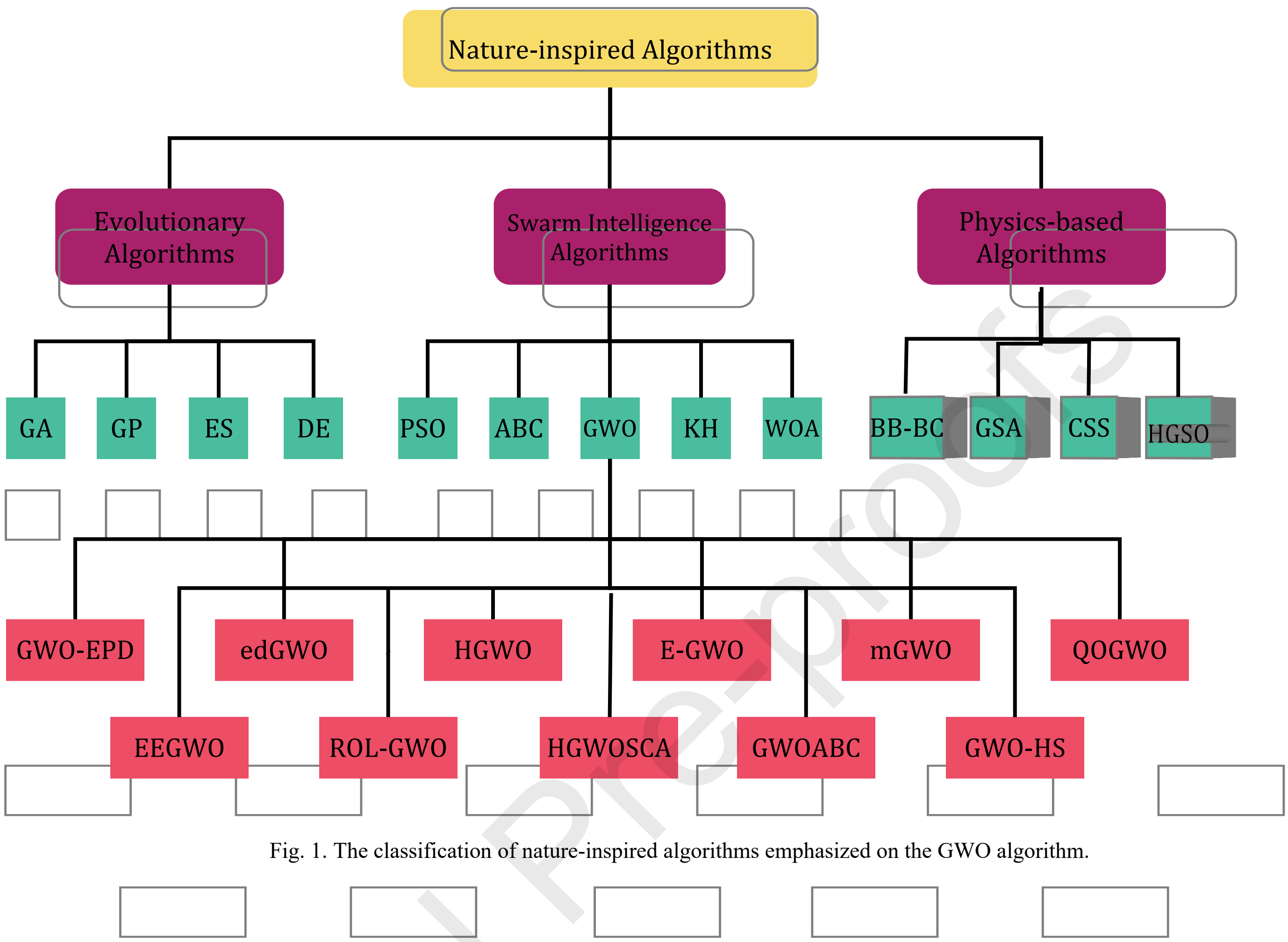

Fig. 1. The classification of nature-inspired algorithms emphasized on the GWO algorithm.

3. GREY WOLF OPTIMIZER (GWO)

The grey wolf optimizer (GWO) algorithm is inspired by the social leadership and hunting behavior of grey wolves in nature. The GWO algorithm considers three leader wolves named α, β, and δ as the best solutions to lead the rest of the wolves named ω wolves toward promising areas in order to find the global solution. The wolf hunting consists of three main steps: encircling, hunting, and attacking the prey.

- Encircling**:** Encircling the prey by the grey wolves can be modeled, as shown in Eqs. (1) and (2).

$$D = \left| C \times X_p(t) - X(t) \right| \tag{1}$$

$$X(t+1) = X_p(t) - A \times D \tag{2}$$

Where X_p is the prey position, X indicates the position vector of a grey wolf, t is the current iteration. C and A are the coefficient vectors calculated by Eqs. (3) and (4).

$$A = 2 \times A \times r_1 - a(t) \tag{3}$$

$$C = 2 \times r_2 \tag{4}$$

Where r_1, r_2 are random vectors in [0,1], and the elements of the vector a are linearly decrease from 2 to 0 over the course of iterations by Eq. (5).

$$a(t) = 2 - (2 \times t) / MaxIter \tag{5}$$

- Hunting**:** To mathematically model wolves' hunting behavior, it is assumed that α, β, and δ have better knowledge about the location of the prey. Therefore, by considering the position of the three best solutions α, β, and δ, the other wolves ω are obliged to follow them. The following Eqs. (6-8) are described the hunting behavior.

$$\begin{aligned} D_\alpha &= \left| C_1 \times X_\alpha - X(t) \right|, \\ D_\beta &= \left| C_2 \times X_\beta - X(t) \right|, \\ D_\delta &= \left| C_3 \times X_\delta - X(t) \right| \end{aligned} \tag{6}$$

Where C_1, C_2, and C_3 are calculated by Eq. (4).

$$\begin{aligned} X_{i1}(t) &= X_\alpha(t) - A_{i1} \times D_\alpha(t), \\ X_{i2}(t) &= X_\beta(t) - A_{i2} \times D_\beta(t), \\ X_{i3}(t) &= X_\delta(t) - A_{i3} \times D_\delta(t) \end{aligned} \tag{7}$$

Where X_α, X_β, and X_δ are the first three best solutions at iteration t, A_1, A_2, and A_3 are calculated as in Eq. (3), and D_α, D_β, and D_δ are defined as Eq. (6).

$$X(t+1) = \frac{X_{i1}(t) + X_{i2}(t) + X_{i3}(t)}{3} \tag{8}$$

- Attacking**:** The hunting process is terminated when the prey stops moving, and wolves start an attack. This can be done mathematically by the value of a which is linearly decreased over the curse of iterations controlling the exploration and exploitation. As shown in Eq. (5), it is updated in each iteration to range from 2 to 0. According to (Emary et al., 2017), half of the iterations are dedicated to the exploration, and with a smooth transition, the other half is assigned to exploitation. In this step, wolves change their positions to any random position between the prey position and their current position.

Detailed flowchart of the GWO algorithm is shown in Fig. 2. The algorithm starts by randomly generating an initial population of wolves within the search space. The fitness function evaluates the wolves' positions. Then the following steps are repeated until the stopping criterion is satisfied. The stopping criterion is to reach the predefined number of iterations (Maxiter). In each iteration, the three first wolves with the best finesses are considered as α, β, and δ. After that, each wolf updates its position with respect to the aforementioned steps encircling, hunting, and attacking the prey. Finally, by repeating these steps, the best location of the prey, which is the α's position, can be located.

Although GWO is simple and applicable for several applications, it suffers from lack of population diversity, the imbalance between the exploitation and exploration, and the premature convergence (Heidari et al., 2017; Long et al., 2018; Lu et al., 2018; Tu et al., 2019a). Furthermore, the position update equation of the GWO is good at exploitation, but it is not sufficient for obtaining a feasible solution.

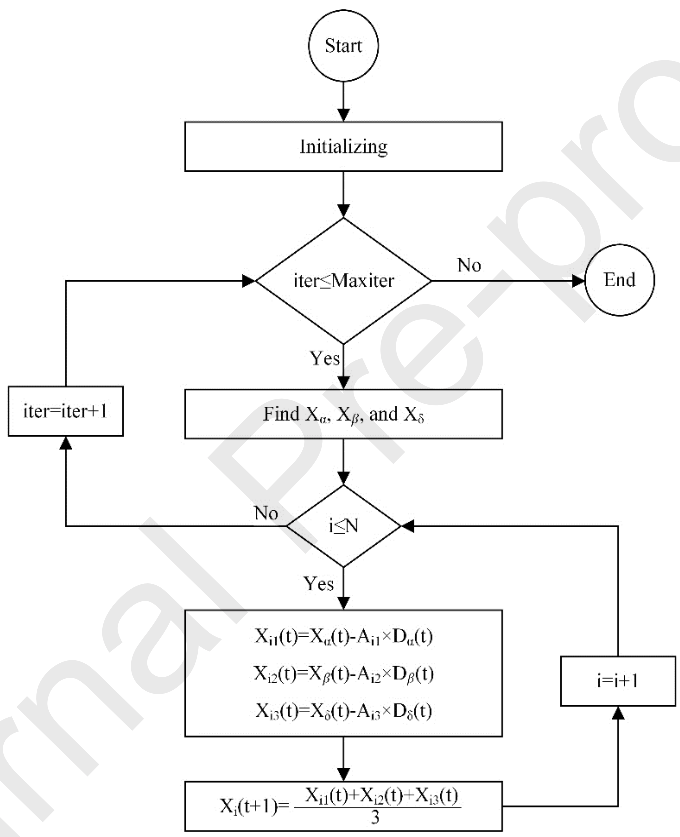

Fig. 2. The flowchart of the GWO algorithm.

4. IMPROVED GREY WOLF OPTIMIZER (I-GWO)

In GWO, α, β, and δ lead ω wolves toward the areas of the search space that are promising for finding the optimal solution. This behavior may lead to entrapment in locally optimal solution. Another side-effect is the reduction of the diversity of the population and cause GWO to fall into the local optimum. To overcome these issues, in this section, an improved grey wolf optimizer (I-GWO) is proposed. The improvements include a new search strategy associated by selecting and updating step, which are indicated in the dashed line border in the flowchart of I-GWO shown in Fig. 3. Then, the I-GWO includes three phases: initializing, movement, and selecting and updating as follows.

Initializing phase: In this phase, N wolves are randomly distributed in the search space in a given range [l_i, u_j] by Eq. (9).

$$X_{ij} = l_j + rand_j[0,1] \times (u_j - l_j), i \in [1, N], j \in [1, D] \tag{9}$$

The position of the i-th wolf in the t-th iteration is represented as a vector of real values $X_i(t)$ = {x_{i1}, x_{i2}, …, x_{iD}}, where D is the dimension number of the problem. The whole population of wolves is stored in a matrix Pop, which has N rows and D columns. The fitness value of X_i (t) is calculated by the fitness function, f (X_i (t)).

Movement phase: In addition to group hunting, individual hunting is another interesting social behavior of grey wolves (MacNulty et al., 2007), which is our motivation to improve the GWO. The I-GWO incorporates an additional movement strategy named dimension learning-based hunting (DLH) search strategy. In DLH, each individual wolf is learned by its neighbors to be another candidate for the new position of X_i (t). The following steps describe how canonical GWO and DLH search strategies generate two different candidates.

The canonical GWO search strategy: As described in Section 3, in GWO, the first three best wolves from Pop are considered as α, β, and δ. After that, the linearly decreased coefficient a, and coefficients A and C are calculated by Eqs. (3-5). Then, the prey encircling is determined by considering the position of X_α, X_β, and X_δ by Eqs. (6 and 7). Finally, the first candidate for the new position of wolf X_i (t) named $X_{i\text{-GWO}}$ (t+1) is calculated by Eq. (8).

Dimension learning-based hunting (DLH) search strategy: In the original GWO, for each wolf, a new position is generated with the help of three leader wolves of the Pop. This way causes that GWO shows slow convergence, the population loses diversity too early, and wolves are trapped in the local optima. To tackle these defects, in the proposed DLH search strategy, individual hunting of wolves is considered that is learned by its neighbors.

In the DLH search strategy, each dimension of the new position of wolf X_i (t) is calculated by Eq. (12) in which this individual wolf is learned by its different neighbors and a randomly selected wolf from Pop. Then, besides $X_{i\text{-GWO}}$ (t+1), the DLH search strategy generates another candidate for the new position of wolf X_i (t) named $X_{i\text{-DLH}}$ (t+1). For doing this, first, a radius $R_i(t)$ is calculated using Euclidean distance between the current position of X_i (t) and the candidate position $X_{i\text{-GWO}}$ (t+1) by Eq. (10).

$$R_i(t) = \left\| X_i(t) - X_{i-GWO}(t+1) \right\| \tag{10}$$

Then, the neighbors of X_i (t) denoted by N_i (t) is constructed by Eq. (11) respected to radius R_i (t), where D_i is Euclidean distance between X_i (t) and X_j (t).

$$N_i(t) = \left\{ X_j(t) \mid D_i\left(X_i(t), X_j(t)\right) \leq R_i(t), X_j(t) \in Pop \right\} \tag{11}$$

Once the neighborhood of X_i (t) is constructed, multi neighbors learning is performed by Eq. (12) where the d-th dimension of $X_{i\text{-}DLH,d}$ (t+1) is calculated by using the d-th dimension of a random neighbor $X_{n,d}$ (t) selected from N_i (t), and a random wolf $X_{r,d}$ (t) from Pop.

$$X_{i-DLH,d}(t+1) = X_{i,d}(t) + rand \times (X_{n,d}(t) - X_{r,d}(t)) \tag{12}$$

Selecting and updating phase: In this phase, first, the superior candidate is selected by comparing the fitness values of two candidates $X_{i\text{-}GWO}$ (t+1) and $X_{i\text{-}DLH}$ (t+1) by Eq. (13).

$$X_i(t+1) = \begin{cases} X_{i-GWO}(t+1), & if\ f(X_{i-GWO}) < f(X_{i-DLH}) \\ X_{i-DLH}(t+1) & otherwise \end{cases} \tag{13}$$

Then, in order to update the new position of X_i (t+1), if the fitness value of the selected candidate is less than X_i (t), X_i (t) is updated by the selected candidate. Otherwise, X_i (t) remains unchanged in the Pop.

Finally, after performing this procedure for all individuals, the counter of iterations (iter) is increased by one, and search can be iterated until the predefined number of iterations (Maxiter) is reached. The pseudo-code of the proposed I-GWO algorithm is shown in Fig. 4.

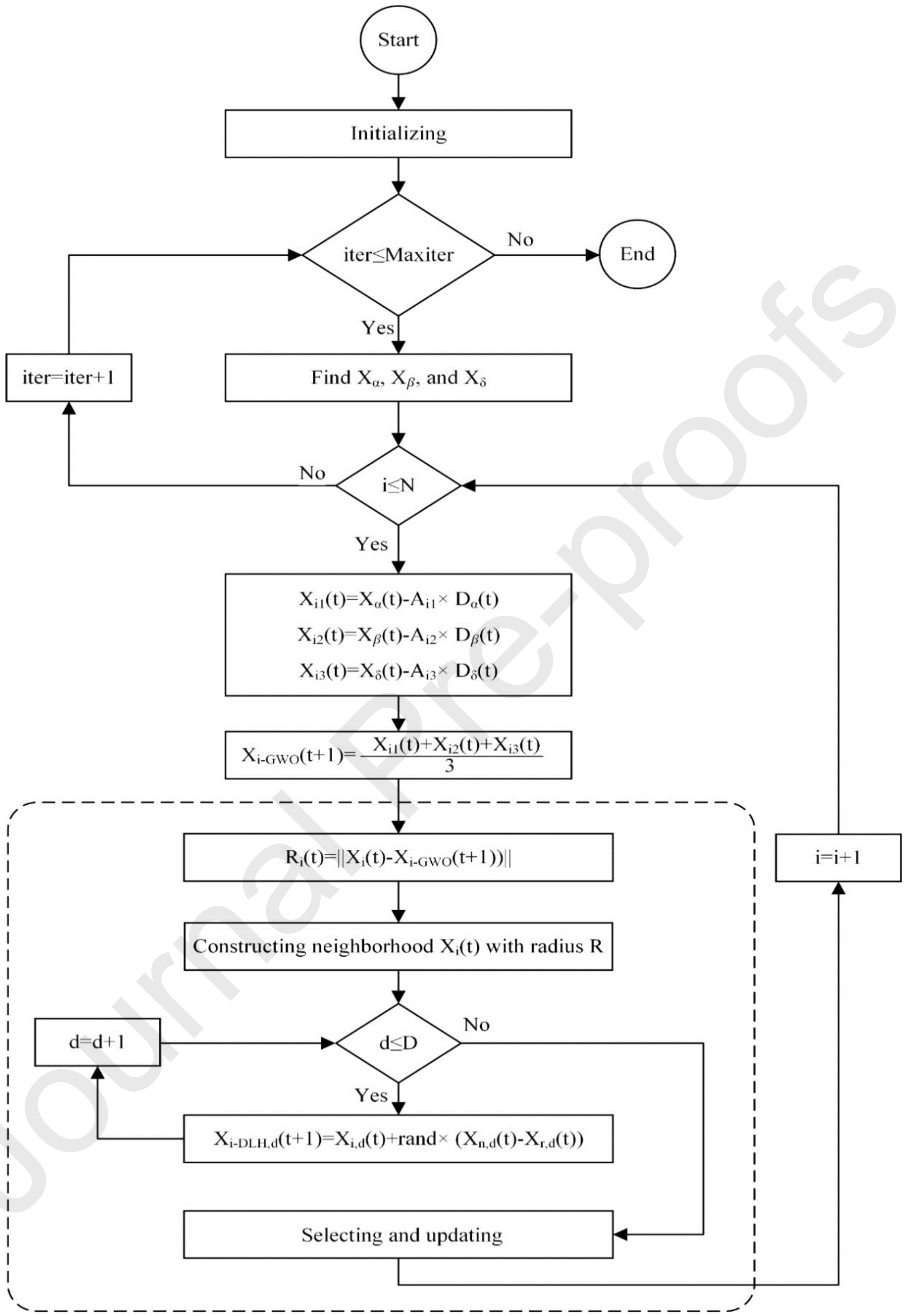

Fig. 3. The flowchart of the I-GWO algorithm.

Algorithm 1: The Improved grey wolf optimizer algorithm (I-GWO)

Input: N, D, Maxiter
Output: The global optimum
Begin
Initializing (Randomly distributing N wolves in the search space and calculating their fitness).
For iter = 2 to Maxiter
    Find X_α, X_β, and X_δ.
    For i = 1 to N
        Computing X_{i1}, X_{i2}, X_{i3} by using Eq. (7).
        Computing $X_{i\text{-}GWO}$ (t+1) by using Eq. (8).
        Calculating R_i (t) by Eq. (10).
        Constructing neighborhood X_i (t) with radius R_i by Eq. (11).
        For d = 1 to D
            $X_{i\text{-}DLH,d}(t+1) = X_{i,d}(t) + rand \times (X_{n,d}(t) - X_{r,d}(t))$
        End for
        Selecting best ($X_{i\text{-}GWO}$ (t+1), $X_{i\text{-}DLH}$ (t+1)).
        Updating Pop.
    End for
End for
Return the global optimum.
End

Fig. 4. The pseudo code of I-GWO.

5. Experimental Evaluation and Results

In this section, the performance of the proposed I-GWO algorithm is evaluated over several test functions by various experiments.

5.1 Benchmark Functions and Experimental Environment

The performance evaluation of the I-GWO was conducted by CEC 2018 benchmark suite that contains 29 test functions (Awad et al.). These test functions include: unimodal (F1, F3), multimodal (F4-F10), hybrid (F11-F20), and composition (F21-F30) functions. All benchmark functions were evaluated with different dimensions of 10, 30, and 50 by 20 independent runs. In each run, the maximum iterations (MaxIter) was set by (D×10000)/N, where D and N are the dimensions of the problem and the population size. The value of N was set to 100, respectively.

All experiments were performed on a CPU, Intel Core(TM) i7-3770 3.4GHz and 8.00 GB RAM, and Matlab R2018a was used for programming. The results are reported based on the fitness discrepancy (error), $f-f^*$ where f is the optimization result obtained by the corresponding algorithm, and f^* is the global optimum. The mean and standard deviation of fitness error were employed to measure the performance of the algorithms. The experimental results are shown in Tables 2-5, in which the bold values show the best-obtained solutions. Moreover, the last three rows of each table denoted “w/t/l” indicate the number of wins (w), ties (t), and losses (l) of each algorithm.

The results of the I-GWO are compared with the state-of-the-art metaheuristic algorithms: PSO (Eberhart et al., 1995), KH (Gandomi et al., 2012), GWO (Mirjalili et al., 2014), WOA (Mirjalili et al., 2016), EEGWO (Long et al., 2018), and HGSO (Hashim et al., 2019). As shown in Table 1, in all experiments, the parameters of the comparative algorithms were the same as the recommended settings in their original works.

Table 1. Parameters settings.

Algs	**Setting**
PSO	c1=c2=2
KH	Vf=0.02, Dmax=0.005, Nmax=0.01
GWO	a was linearly decreased from 2 to 0
WOA	a = [2 0], b = 1
EEGWO	b1=0.1, b2=0.9, μ=1.5, ainitial =2, afinal=0
HGSO	Cluster number=5, M1=0.1, M2=0.2, α=β=K=1, l1=0.005, l2=100, l3=0.01
I-GWO	a was linearly decreased from 2 to 0

5.2 Exploration and Exploitation Evaluation

The unimodal test functions are suitable for verifying the exploitation ability to find the optimal solution. On the other hand, the multimodal functions with many local minima can test the ability of the I-GWO in the exploration and also in the local optima avoidance.

Inspecting the results in Table 2, it is evident that the I-GWO algorithm is able to provide very competitive results on unimodal test functions, especially significantly showed improved results on F3 for all dimensions. Therefore, it can be concluded that the I-GWO algorithm exploits around the optimum solution more effective than the GWO. According to the results reported in Table 3, I-GWO can provide superior results on the multimodal functions for different dimensions 10, 30, and 50. The experiment is performed on F4-F10, in which the difficulty increases proportional to the number of dimensions. These results show that the proposed I-GWO algorithm is competitive in terms of exploration.

5.3 Evaluation of Local Optima Avoidance

As discussed above, multi-modal and composite test functions test the exploratory behavior of an algorithm. Therefore, the local optima avoidance of the I-GWO can be observed and demonstrated by applying it to such problems. Moreover, the balance between exploitation and exploration can concurrently be benchmarked by these functions.

The results in Table 4 show that I-GWO is superior on all hybrid functions for three different dimensions 10, 30, and 50. Furthermore, Table 5 shows the obtained solutions for solving composition functions by I-GWO and other algorithms, where I-GWO outperforms all other algorithms. As the results demonstrate, I-GWO shows a good balance between exploration and exploitation that results in high local optima avoidance.

5.4 Convergence Evaluation

In the following experiments, the convergence behavior of I-GWO is evaluated and compared by the competitive algorithms. Fig. 5 shows the convergence curves over some selected functions of unimodal and multimodal functions, and Fig. 6 on hybrid and composition functions on dimensions 10, 30, and 50. In these figures, the convergence curves are plotted by the mean of the best solution in each iteration over 20 runs.

The curves show that the I-GWO has high fluctuations in the initial iterations and low variations in the last iterations. The descending trend of curves shows the wolves of the population are collaborating to improve results by updating their position to a better one as the iteration number increases. Then, in all curves, the I-GWO shows better convergence behavior, which can be concluded that it strikes a balance between the exploration and exploitation in the course of iteration more than the opponent algorithms.

Table 2. The comparison of obtained solutions for unimodal functions.

F	D	Index	PSO (1997)	KH (2012)	GWO (2014)	WOA (2016)	EEGWO (2018)	HGSO (2019)	I-GWO
F1	10	Mean	4.9552E+08	**4.8079E+02**	4.3804E+06	2.3203E+05	1.1425E+10	5.6412E+08	3.6746E+03
		STD	1.7245E+08	5.1488E+02	1.1538E+07	4.3811E+05	2.0654E+09	2.5186E+08	1.8625E+03
	30	Mean	1.5368E+10	**1.5947E+04**	8.4874E+08	3.2594E+06	5.9660E+10	1.5216E+10	3.3657E+04
		STD	1.8406E+09	1.4353E+04	7.7867E+08	2.3799E+06	4.5667E+09	3.2405E+09	1.0010E+04
	50	Mean	3.7785E+10	2.3058E+05	4.0637E+09	9.7205E+06	1.1551E+11	3.9533E+10	**2.4101E+03**
		STD	3.8007E+09	1.7007E+05	2.1433E+09	1.1581E+07	4.5644E+09	8.5861E+09	1.1730E+03
F3	10	Mean	3.0546E+03	9.0070E+02	5.0896E+02	2.1389E+02	1.4228E+04	7.3095E+02	**7.4030E-03**
		STD	7.4726E+02	1.1341E+03	1.9473E+03	2.5377E+02	2.7229E+03	3.0711E+02	2.6115E-03
	30	Mean	4.9095E+04	4.5347E+04	2.8332E+04	1.6691E+05	8.8839E+04	3.8449E+04	**3.3321E+02**
		STD	5.5916E+03	1.4117E+04	8.0961E+03	7.7951E+04	4.8504E+03	7.1682E+03	1.5508E+02
	50	Mean	1.0748E+05	1.1527E+05	6.5679E+04	7.4951E+04	1.4113E+06	1.3864E+05	**2.4203E+03**
		STD	1.9222E+04	2.2298E+04	1.5390E+04	2.9689E+04	3.1911E+06	8.3860E+03	1.4764E+03
Rank	10	w/t/l	0/0/2	1/0/1	0/0/2	0/0/2	0/0/2	0/0/2	1/0/1
	30	w/t/l	0/0/2	1/0/1	0/0/2	0/0/2	0/0/2	0/0/2	1/0/1
	50	w/t/l	0/0/2	0/0/2	0/0/2	0/0/2	0/0/2	0/0/2	2/0/0

Table 3. The comparison of obtained solutions for multimodal functions.

F	D	Index	PSO (1997)	KH (2012)	GWO (2014)	WOA (2016)	EEGWO (2018)	HGSO (2019)	I-GWO
F4	10	Mean	4.1550E+01	5.9099E+00	1.5966E+01	2.2290E+01	1.1517E+03	4.6494E+01	**2.0588E+00**
		STD	7.5025E+00	2.4220E+00	1.9469E+01	3.5023E+01	4.0231E+02	1.2794E+01	5.1591E-01
	30	Mean	9.0785E+02	9.8725E+01	1.4472E+02	1.5935E+02	1.8550E+04	1.6256E+03	**8.6678E+01**
		STD	1.5718E+02	2.0332E+01	3.1260E+01	4.2152E+01	2.8135E+03	4.4775E+02	3.5732E+00
	50	Mean	3.3841E+03	1.5434E+02	4.1183E+02	2.8050E+02	3.9474E+04	7.7108E+03	**9.7619E+01**
		STD	4.2116E+02	5.0563E+01	1.6721E+02	5.5343E+01	2.7136E+03	1.7846E+03	2.7134E+01
F5	10	Mean	3.4883E+01	2.6914E+01	1.3829E+01	4.8430E+01	1.2171E+02	4.9689E+01	**1.1319E+01**
		STD	2.8317E+00	7.1345E+00	8.1265E+00	1.4771E+01	1.1238E+01	5.3829E+00	5.8106E+00
	30	Mean	2.5382E+02	1.3714E+02	9.1704E+01	2.6365E+02	4.6764E+02	3.0625E+02	**5.2225E+01**
		STD	1.9973E+01	3.1379E+01	3.3006E+01	5.4562E+01	1.7271E+01	1.7137E+01	4.7202E+01
	50	Mean	5.3118E+02	2.5648E+02	1.8244E+02	4.0646E+02	7.4489E+02	5.4451E+02	**6.6577E+01**
		STD	1.7559E+01	3.1050E+01	3.7258E+01	7.6040E+01	2.6318E+01	2.8022E+01	1.6360E+01
F6	10	Mean	1.8545E+01	5.4033E+00	1.7018E-01	2.5601E+01	6.6527E+01	2.4253E+01	**2.9214E-02**
		STD	1.5853E+00	6.8054E+00	2.1016E-01	1.2741E+01	5.4502E+00	4.3041E+00	6.6504E-03
	30	Mean	4.0878E+01	3.5478E+01	4.0997E+00	6.6469E+01	1.0399E+02	6.4811E+01	**6.7371E-02**
		STD	2.7951E+00	9.0552E+00	1.8595E+00	8.9686E+00	5.0750E+00	6.3506E+00	8.9037E-03
	50	Mean	5.3056E+01	5.1336E+01	1.0505E+01	7.6778E+01	1.1476E+02	8.2391E+01	**7.1289E-02**
		STD	3.6735E+00	6.4554E+00	3.9121E+00	9.8048E+00	3.0641E+00	4.7383E+00	9.9378E-03
F7	10	Mean	1.2052E+02	**2.1001E+01**	2.6981E+01	7.4922E+01	1.4219E+02	6.6314E+01	2.2449E+01
		STD	1.1907E+01	5.3417E+00	7.0974E+00	2.3504E+01	9.3820E+00	7.3019E+00	6.8446E+00
	30	Mean	8.2739E+02	1.3260E+02	1.2569E+02	4.8959E+02	7.5997E+02	4.0094E+02	**1.1676E+02**
		STD	9.6812E+01	2.6368E+01	3.1442E+01	1.0447E+02	4.0096E+01	3.3183E+01	5.8790E+01
	50	Mean	2.0107E+03	3.6249E+02	3.3471E+02	9.9487E+02	1.3834E+03	8.2904E+02	**1.3793E+02**
		STD	2.5031E+02	5.6609E+01	8.5908E+01	7.3579E+01	4.1122E+01	6.4260E+01	5.4672E+01
F8	10	Mean	4.5376E+01	1.6318E+01	1.2545E+01	4.0125E+01	7.1595E+01	3.2076E+01	**6.7485E+00**
		STD	7.4496E+00	6.9565E+00	6.2583E+00	2.1323E+01	6.0356E+00	3.3007E+00	5.1040E+00
	30	Mean	2.6495E+02	1.0871E+02	7.6365E+01	2.0831E+02	3.7925E+02	2.5330E+02	**3.5734E+01**
		STD	1.2979E+01	1.6888E+01	1.4647E+01	4.7504E+01	1.8265E+01	1.3562E+01	3.2362E+01
	50	Mean	5.2527E+02	2.8378E+02	1.9575E+02	4.1536E+02	7.6900E+02	5.7140E+02	**5.8658E+01**
		STD	1.8131E+01	4.7747E+01	3.1627E+01	7.3544E+01	2.1590E+01	2.7480E+01	1.0278E+01
F9	10	Mean	3.4584E+02	8.5193E+00	2.2349E+00	4.7914E+02	1.0849E+03	9.7986E+01	**7.3741E-04**
		STD	8.8971E+01	2.2798E+01	4.2107E+00	3.5934E+02	2.3980E+02	2.4342E+01	3.6844E-04
	30	Mean	5.0939E+03	2.3082E+03	5.4051E+02	5.9263E+03	1.2726E+04	4.9770E+03	**9.1159E-02**
		STD	5.1996E+02	6.6176E+02	4.4070E+02	1.9896E+03	1.0372E+03	9.9041E+02	4.3756E-02
	50	Mean	1.3578E+04	9.4078E+03	4.5701E+03	1.9529E+04	4.4149E+04	2.6082E+04	**3.3338E-01**
		STD	1.2870E+03	1.3620E+03	3.2583E+03	4.9687E+03	2.7749E+03	2.0188E+03	1.1790E-01
F10	10	Mean	1.1999E+03	9.9351E+02	5.3932E+02	1.0556E+03	2.3242E+03	1.3586E+03	**8.5738E+01**
		STD	1.7606E+02	2.8832E+02	1.9411E+02	2.9700E+02	1.6881E+02	1.8230E+02	8.7926E+01
	30	Mean	6.9815E+03	4.0559E+03	**2.8508E+03**	5.2298E+03	8.4006E+03	5.7170E+03	4.0107E+03
		STD	1.9678E+02	4.3583E+02	5.5476E+02	6.3974E+02	4.6515E+02	3.4356E+02	2.2022E+03
	50	Mean	1.3403E+04	6.6518E+03	**5.6339E+03**	9.0709E+03	1.5377E+04	1.1703E+04	5.8217E+03
		STD	5.1086E+02	8.5488E+02	6.9439E+02	1.3014E+03	3.6916E+02	6.8363E+02	3.6995E+03
Rank	10	w/t/l	0/0/7	1/0/6	0/0/7	0/0/7	0/0/7	0/0/7	6/0/1
	30	w/t/l	0/0/7	0/0/7	1/0/6	0/0/7	0/0/7	0/0/7	6/0/1
	50	w/t/l	0/0/7	0/0/7	1/0/6	0/0/7	0/0/7	0/0/7	6/0/1

Table 4. The comparison of obtained solutions for hybrid functions.

F	D	Index	PSO (1997)	KH (2012)	GWO (2014)	WOA (2016)	EEGWO (2018)	HGSO (2019)	I-GWO
F11	10	Mean	1.4001E+02	4.0922E+01	2.2286E+01	1.0023E+02	3.4725E+03	1.3039E+02	**1.9082E+00**
		STD	4.3362E+01	2.3029E+01	1.2791E+01	5.8982E+01	2.1210E+03	4.4290E+01	9.3551E-01
	30	Mean	1.6835E+07	1.0974E+06	9.1104E+05	2.5032E+06	7.6811E+08	6.4785E+06	**7.0127E+03**
		STD	8.2779E+06	9.3630E+05	1.8098E+06	2.6979E+06	4.0856E+08	2.9613E+06	3.7513E+03
	50	Mean	5.4664E+04	9.7718E+03	9.7509E+03	1.4700E+04	3.3990E+07	1.6699E+04	**3.6799E+02**
		STD	4.5698E+04	6.4749E+03	6.5840E+03	1.1075E+04	2.7177E+07	8.3235E+03	1.1089E+02
F12	10	Mean	1.2424E+02	8.0174E+02	9.8408E+02	1.8732E+02	1.4020E+05	5.3937E+02	**4.0000E+01**
		STD	3.1936E+01	1.4209E+03	1.6069E+03	1.7130E+02	2.4240E+05	5.4248E+02	4.9872E+00
	30	Mean	9.9546E+02	2.9681E+03	1.4771E+03	2.7740E+03	7.7473E+04	2.3338E+03	**1.7248E+01**
		STD	9.2609E+02	3.4791E+03	1.7152E+03	3.4991E+03	1.1742E+05	1.2329E+03	6.3856E+00
	50	Mean	8.0013E+01	3.1772E+02	4.4638E+01	2.0043E+02	7.0614E+02	2.1346E+02	**2.6329E+00**
		STD	3.5258E+01	1.2186E+02	3.9803E+01	1.4516E+02	1.2800E+02	6.2532E+01	1.1602E+00
F13	10	Mean	8.1247E+01	5.7146E+01	5.1096E+01	9.0137E+01	2.3281E+02	7.2962E+01	**3.2255E+01**
		STD	9.8471E+00	1.5988E+01	2.6040E+01	4.3818E+01	6.8609E+01	7.0434E+00	3.8678E+00
	30	Mean	5.1792E+04	9.3726E+03	2.6070E+04	1.8298E+04	2.9406E+08	2.0660E+05	**1.1351E+03**
		STD	4.4385E+04	7.8346E+03	1.5208E+04	1.3315E+04	2.2101E+08	1.6105E+05	7.8059E+02
	50	Mean	5.0106E+02	1.2508E+03	2.2156E+03	1.4674E+04	5.0464E+06	4.1782E+03	**1.8170E+01**
		STD	3.4457E+02	2.0934E+03	4.4939E+03	1.4334E+04	5.8048E+06	3.5407E+03	4.2825E+00
F14	10	Mean	8.7894E+01	1.3709E+02	5.8560E+01	1.1318E+02	3.3494E+02	1.1526E+02	**2.1170E+01**
		STD	1.6203E+01	7.0946E+01	5.0893E+01	5.1960E+01	5.6744E+01	3.5694E+01	7.4368E+00
	30	Mean	1.8339E+03	4.0186E+02	3.1822E+02	3.2385E+02	1.0672E+04	1.4416E+03	**9.5843E+00**
		STD	3.5158E+02	1.8390E+02	2.0078E+02	7.5186E+01	2.4161E+03	5.8356E+02	2.9359E+00
	50	Mean	1.1599E+09	2.8276E+06	3.1509E+07	4.2886E+07	1.6558E+10	1.2722E+09	**2.9215E+05**
		STD	2.5929E+08	1.7654E+06	2.8507E+07	2.9422E+07	1.5347E+09	5.3240E+08	1.5523E+05
F15	10	Mean	4.1458E+08	3.7432E+04	8.6219E+04	1.1017E+05	1.7070E+10	4.3619E+08	**2.8438E+04**
		STD	1.1488E+08	1.8027E+04	6.9184E+04	4.5900E+04	3.2409E+09	1.3734E+08	1.2312E+04
	30	Mean	1.4824E+05	3.4480E+05	1.1374E+05	9.6327E+05	1.4347E+07	4.4603E+05	**6.1402E+02**
		STD	6.0811E+04	4.5784E+05	2.8220E+05	1.5151E+06	9.4128E+06	2.2081E+05	3.2728E+02
	50	Mean	7.6400E+07	1.6365E+04	2.3397E+05	1.0406E+05	5.8632E+08	3.2849E+06	**2.0521E+03**
		STD	3.5243E+07	6.4037E+03	5.8191E+05	1.0358E+05	2.5159E+08	1.6386E+06	1.1282E+03
F16	10	Mean	1.7507E+03	1.1560E+03	6.9067E+02	1.8136E+03	5.8074E+03	2.0956E+03	**1.3953E+02**
		STD	1.4589E+02	2.8099E+02	2.1689E+02	5.0735E+02	8.6662E+02	1.3463E+02	1.0998E+02
	30	Mean	8.0540E+02	4.6158E+02	2.3266E+02	7.4875E+02	5.4397E+03	8.0765E+02	**6.8159E+01**
		STD	1.3590E+02	2.1256E+02	1.2344E+02	2.5902E+02	4.2328E+03	1.2894E+02	2.6880E+01
	50	Mean	3.3126E+06	3.9365E+05	6.5272E+05	1.8884E+06	1.5449E+08	2.2317E+06	**6.7323E+04**
		STD	1.7577E+06	4.4470E+05	6.8499E+05	1.9841E+06	8.1917E+07	1.3974E+06	2.5864E+04
F17	10	Mean	9.0888E+07	1.2013E+05	5.1055E+05	2.6078E+06	1.4092E+09	9.3449E+06	**2.7343E+03**
		STD	4.5084E+07	1.2779E+05	4.7986E+05	2.8071E+06	6.1333E+08	3.3162E+06	2.1585E+03
	30	Mean	6.1882E+02	5.7612E+02	3.9264E+02	6.4772E+02	1.3288E+03	5.9116E+02	**9.1121E+01**
		STD	9.4183E+01	2.1705E+02	1.7362E+02	1.9879E+02	1.2914E+02	6.8158E+01	5.5552E+01
	50	Mean	5.2347E+03	3.5616E+03	1.2273E+03	4.9999E+02	2.4395E+04	4.8893E+03	**5.6194E+01**
		STD	8.8606E+02	1.6932E+03	9.4525E+02	1.0180E+02	1.8045E+03	1.2530E+03	5.2159E+00
F18	10	Mean	7.5466E+09	1.0633E+07	4.7401E+08	1.8504E+08	9.1981E+10	1.5669E+10	**4.1449E+06**
		STD	8.4065E+08	9.2502E+06	6.1872E+08	1.0098E+08	9.5207E+09	3.8922E+09	1.0876E+06
	30	Mean	2.3524E+09	4.8033E+04	8.3780E+07	1.5515E+05	5.2758E+10	2.3606E+09	**3.2714E+04**
		STD	5.8351E+08	2.2937E+04	1.0390E+08	1.1176E+05	7.1395E+09	9.7418E+08	7.5405E+03
	50	Mean	1.1125E+06	3.5533E+05	4.7498E+05	5.5527E+05	1.8688E+08	3.3990E+06	**1.1876E+04**
		STD	4.8884E+05	3.4115E+05	7.0945E+05	4.0930E+05	8.9725E+07	1.2786E+06	7.0563E+03
F19	10	Mean	8.1842E+08	2.1156E+04	3.5922E+06	8.3442E+04	1.0452E+10	2.3107E+08	**1.0755E+04**
		STD	2.3455E+08	7.1365E+03	7.4502E+06	6.3157E+04	2.1011E+09	5.4977E+07	4.0019E+03
	30	Mean	3.7921E+03	1.7398E+03	1.2685E+03	3.3390E+03	9.5715E+03	2.9663E+03	**3.3612E+02**
		STD	2.3905E+02	3.1262E+02	3.9197E+02	7.4499E+02	1.2196E+03	4.2387E+02	9.1931E+01
	50	Mean	3.7743E+03	1.6385E+03	9.9761E+02	2.3952E+03	1.4241E+04	2.1140E+03	**2.9335E+02**
		STD	2.9998E+02	2.8177E+02	2.4768E+02	3.4685E+02	3.7662E+03	2.2858E+02	1.1301E+02
F20	10	Mean	1.2796E+07	2.6213E+06	2.6971E+06	5.2563E+06	2.7328E+08	9.1165E+06	**2.7002E+05**
		STD	3.3769E+06	1.5811E+06	3.9582E+06	5.0881E+06	7.9547E+07	2.6029E+06	1.2415E+05
	30	Mean	3.9738E+08	1.6615E+05	3.1386E+06	2.2840E+06	6.1205E+09	1.6702E+08	**1.1622E+04**
		STD	1.0841E+08	1.1796E+05	7.3260E+06	1.9068E+06	1.3312E+09	6.2864E+07	7.5368E+03
	50	Mean	1.7928E+03	1.3268E+03	8.0327E+02	1.6588E+03	2.6333E+03	1.4016E+03	**2.3013E+02**
		STD	1.7961E+02	3.2515E+02	2.4705E+02	2.3094E+02	2.6268E+02	2.1354E+02	2.2358E+02
Rank	10	w/t/l	0/0/10	0/0/10	0/0/10	0/0/10	0/0/10	0/0/10	10/0/0
	30	w/t/l	0/0/10	0/0/10	0/0/10	0/0/10	0/0/10	0/0/10	10/0/0
	50	w/t/l	0/0/10	0/0/10	0/0/10	0/0/10	0/0/10	0/0/10	10/0/0

Table 5. The comparison of obtained solutions for composition functions.

F	D	Index	PSO (1997)	KH (2012)	GWO (2014)	WOA (2016)	EEGWO (2018)	HGSO (2019)	I-GWO
F21	10	Mean	2.1235E+02	1.2758E+02	2.0734E+02	2.0710E+02	3.0949E+02	1.5193E+02	**1.2590E+02**
		STD	5.2750E+01	4.6902E+01	2.2944E+01	5.9152E+01	3.6292E+01	3.8124E+01	4.6003E+01
	30	Mean	4.4529E+02	3.1221E+02	2.7619E+02	4.4932E+02	7.0984E+02	4.6696E+02	**2.3172E+02**
		STD	1.1342E+01	2.1806E+01	3.2223E+01	6.4638E+01	4.6974E+01	2.4995E+01	2.7217E+01
	50	Mean	7.0671E+02	4.3856E+02	3.7305E+02	7.5783E+02	1.2437E+03	8.2029E+02	**2.6104E+02**
		STD	1.4275E+01	3.3400E+01	2.6742E+01	9.7087E+01	8.3936E+01	3.3338E+01	7.1552E+00
F22	10	Mean	1.9240E+02	9.4378E+01	1.0426E+02	1.1416E+02	1.0923E+03	1.8251E+02	**9.2002E+01**
		STD	1.9585E+01	2.2969E+01	2.8976E+00	1.7460E+01	2.4214E+02	3.5601E+01	3.2905E+01
	30	Mean	5.4983E+03	1.0039E+02	2.1293E+03	3.7227E+03	7.9868E+03	1.7244E+03	**1.0014E+02**
		STD	2.5154E+03	5.0800E-01	1.5304E+03	2.2536E+03	4.0455E+02	3.6545E+02	2.5329E-02
	50	Mean	1.3486E+04	8.2149E+03	6.1380E+03	9.6954E+03	1.5912E+04	9.2970E+03	**5.2825E+03**
		STD	4.1727E+02	9.2758E+02	6.9282E+02	1.2223E+03	5.5627E+02	2.4773E+03	4.6105E+03
F23	10	Mean	3.3048E+02	3.3272E+02	3.1352E+02	3.4121E+02	5.0710E+02	3.6786E+02	**3.0869E+02**
		STD	3.9686E+00	1.0414E+01	7.2011E+00	1.3550E+01	3.8294E+01	1.0516E+01	4.9282E+00
	30	Mean	5.8921E+02	5.9042E+02	4.3866E+02	7.0604E+02	1.5455E+03	7.8511E+02	**3.7246E+02**
		STD	1.1877E+01	5.5598E+01	3.9338E+01	7.4467E+01	1.9719E+02	4.7525E+01	1.0694E+01
	50	Mean	9.4253E+02	1.0638E+03	6.2448E+02	1.2849E+03	2.7164E+03	1.3454E+03	**4.7054E+02**
		STD	1.3271E+01	1.0194E+02	3.6534E+01	1.5258E+02	2.4704E+02	1.2694E+02	1.4259E+01
F24	10	Mean	3.5465E+02	2.6084E+02	3.4260E+02	3.1616E+02	5.2722E+02	**1.6520E+02**	3.2815E+02
		STD	3.5670E+01	1.2152E+02	9.4487E+00	1.2169E+02	6.4627E+01	5.1764E+01	3.8067E+01
	30	Mean	6.3573E+02	6.9887E+02	4.9881E+02	7.6078E+02	1.8179E+03	8.6643E+02	**4.4879E+02**
		STD	9.4855E+00	7.2409E+01	4.5834E+01	8.5578E+01	1.6894E+02	6.2148E+01	2.7487E+01
	50	Mean	9.4337E+02	1.2321E+03	6.9264E+02	1.2749E+03	3.1577E+03	1.5282E+03	**5.8709E+02**
		STD	1.5685E+01	1.4462E+02	6.5735E+01	1.3829E+02	3.0017E+02	1.1764E+02	8.5068E+01
F25	10	Mean	4.6391E+02	4.2856E+02	4.3590E+02	4.4375E+02	9.9501E+02	4.4958E+02	**3.9786E+02**
		STD	9.5074E+00	2.2570E+01	1.5001E+01	3.1465E+01	1.0784E+02	1.1351E+01	1.7144E-01
	30	Mean	1.3111E+03	4.1950E+02	4.4925E+02	4.4409E+02	3.2724E+03	8.2023E+02	**3.8596E+02**
		STD	1.3594E+02	2.0789E+01	3.1003E+01	2.9979E+01	4.7606E+02	6.4810E+01	1.3491E+00
	50	Mean	3.3946E+03	5.9059E+02	8.9866E+02	6.2678E+02	1.3464E+04	3.9382E+03	**5.1391E+02**
		STD	3.4851E+02	2.4181E+01	1.8009E+02	4.9376E+01	6.3205E+02	5.4780E+02	1.2911E+01
F26	10	Mean	4.3303E+02	4.1323E+02	4.0041E+02	6.0399E+02	1.7182E+03	5.5437E+02	**2.8520E+02**
		STD	6.9696E+01	2.8364E+02	2.9728E+02	3.5628E+02	2.9341E+02	6.2507E+01	6.6198E+01
	30	Mean	3.8725E+03	3.2774E+03	1.8317E+03	4.7373E+03	9.4136E+03	4.4440E+03	**1.0069E+03**
		STD	9.3254E+01	1.3916E+03	2.1218E+02	1.0434E+03	6.3189E+02	3.7227E+02	2.6527E+02
	50	Mean	6.6182E+03	7.3964E+03	3.2515E+03	1.0668E+04	1.5377E+04	8.6352E+03	**1.6239E+03**
		STD	1.5700E+02	8.6871E+02	4.7688E+02	1.6055E+03	5.1076E+02	1.1106E+03	1.1502E+02
F27	10	Mean	4.0397E+02	4.2650E+02	3.9377E+02	4.1310E+02	6.4793E+02	4.2209E+02	**3.8941E+02**
		STD	1.4062E+00	2.6446E+01	2.8834E+00	2.4058E+01	9.1309E+01	1.2602E+01	2.0380E-01
	30	Mean	6.1325E+02	7.0524E+02	5.4047E+02	6.4268E+02	2.5575E+03	5.0001E+02	**4.8638E+02**
		STD	2.1085E+01	8.4027E+01	2.1716E+01	7.2930E+01	4.0392E+02	8.2260E-05	8.5949E+00
	50	Mean	9.9588E+02	1.7002E+03	7.8897E+02	1.3721E+03	5.6369E+03	**5.0001E+02**	5.0297E+02
		STD	6.4300E+01	2.9619E+02	7.0112E+01	2.0665E+02	7.9640E+02	1.5976E-04	4.9578E+00
F28	10	Mean	5.0817E+02	4.6049E+02	5.6252E+02	5.4985E+02	1.0772E+03	4.7703E+02	**3.0009E+02**
		STD	1.1816E+02	1.4265E+02	9.0391E+01	1.7947E+02	1.2426E+02	5.2978E+01	2.0012E-02
	30	Mean	1.0026E+03	4.4718E+02	5.5470E+02	4.9063E+02	5.0379E+03	9.1872E+02	**4.0427E+02**
		STD	6.6812E+01	2.6603E+01	6.0837E+01	2.2024E+01	4.8607E+02	4.5633E+02	5.9014E+00
	50	Mean	1.9208E+03	5.3987E+02	1.1505E+03	6.3395E+02	1.1573E+04	3.4582E+03	**4.6069E+02**
		STD	3.9314E+02	3.8710E+01	3.4736E+02	5.2765E+01	8.6756E+02	1.6013E+03	5.4326E-01
F29	10	Mean	2.9147E+02	3.2573E+02	2.6670E+02	3.6804E+02	8.7217E+02	3.4429E+02	**2.4973E+02**
		STD	2.1778E+01	5.0211E+01	3.8435E+01	6.8970E+01	9.1058E+01	2.1376E+01	7.0082E+00
	30	Mean	1.5382E+03	1.2753E+03	7.9601E+02	1.9230E+03	6.1276E+03	1.3717E+03	**4.5603E+02**
		STD	1.5525E+02	2.8062E+02	1.6964E+02	3.5050E+02	1.5196E+03	2.6198E+02	1.6780E+01
	50	Mean	3.4773E+03	2.3968E+03	1.3255E+03	3.7718E+03	2.1121E+05	3.8380E+03	**4.2848E+02**
		STD	2.8301E+02	5.3119E+02	2.7462E+02	8.3557E+02	1.3309E+05	1.1030E+03	3.6958E+01
F30	10	Mean	9.2521E+05	1.0423E+06	6.7964E+05	2.8394E+05	4.8238E+07	5.5263E+04	**1.7953E+03**
		STD	9.7081E+05	1.6396E+06	8.1531E+05	4.2661E+05	2.6499E+07	1.0962E+05	6.0940E+02
	30	Mean	5.2766E+07	1.9137E+06	4.9767E+06	9.9049E+06	2.3309E+09	7.4541E+07	**1.9199E+04**
		STD	1.9307E+07	1.4899E+06	4.9154E+06	6.4555E+06	8.4450E+08	1.9778E+07	4.0941E+03
	50	Mean	7.1365E+08	4.4754E+07	7.5521E+07	8.8362E+07	9.5078E+09	5.8408E+08	**1.0089E+06**
		STD	1.9544E+08	2.1263E+07	2.1339E+07	2.9338E+07	1.8309E+09	1.3743E+08	1.0281E+05
Rank	10	w/t/l	0/0/10	0/0/10	0/0/10	0/0/10	0/0/10	1/0/9	9/0/1
	30	w/t/l	0/0/10	0/0/10	0/0/10	0/0/10	0/0/10	0/0/10	10/0/0
	50	w/t/l	0/0/10	0/0/10	0/0/10	0/0/10	0/0/10	1/0/9	9/0/1

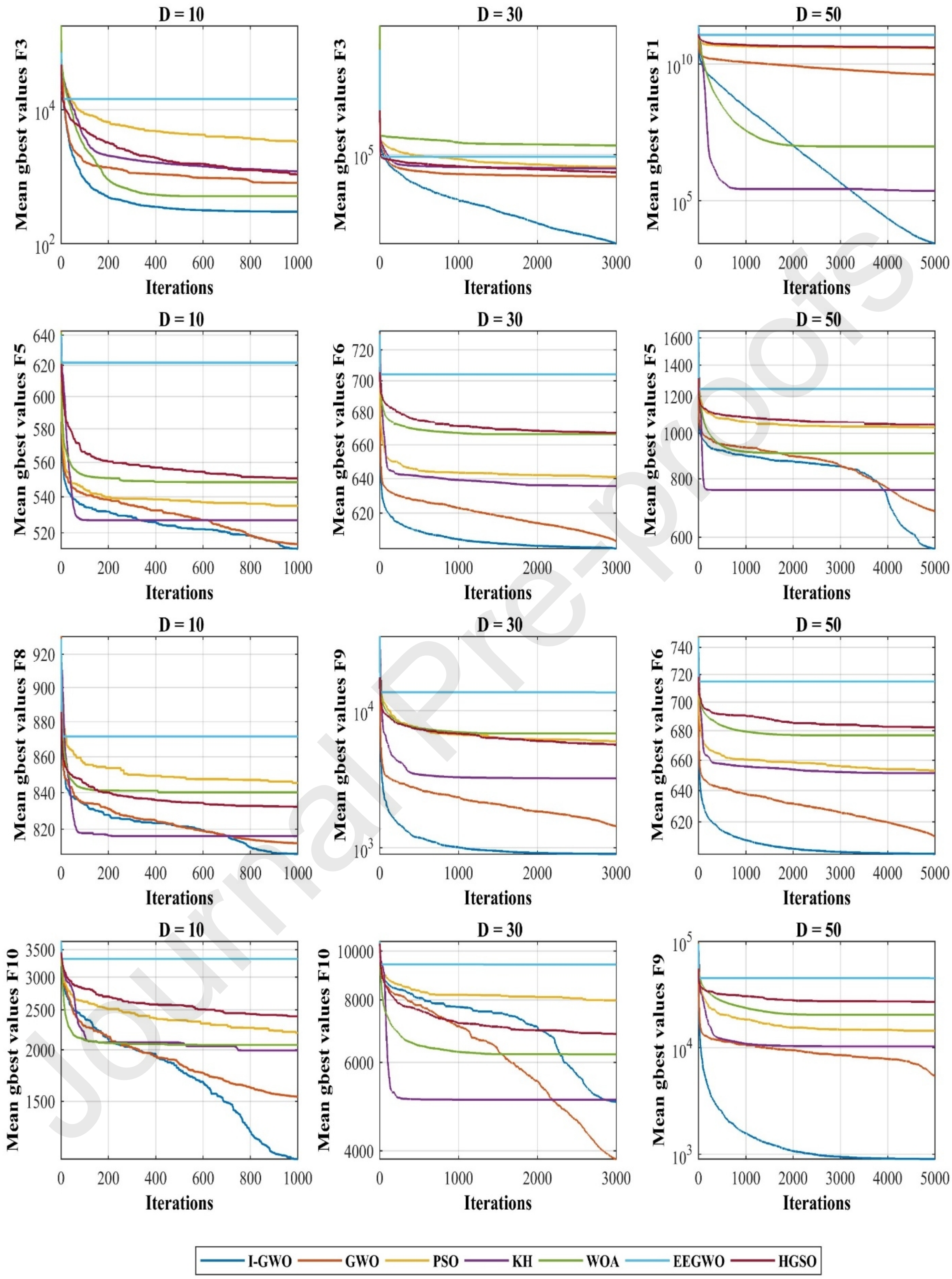

Fig. 5. The convergence curves of unimodal and multimodal functions with different dimensions.

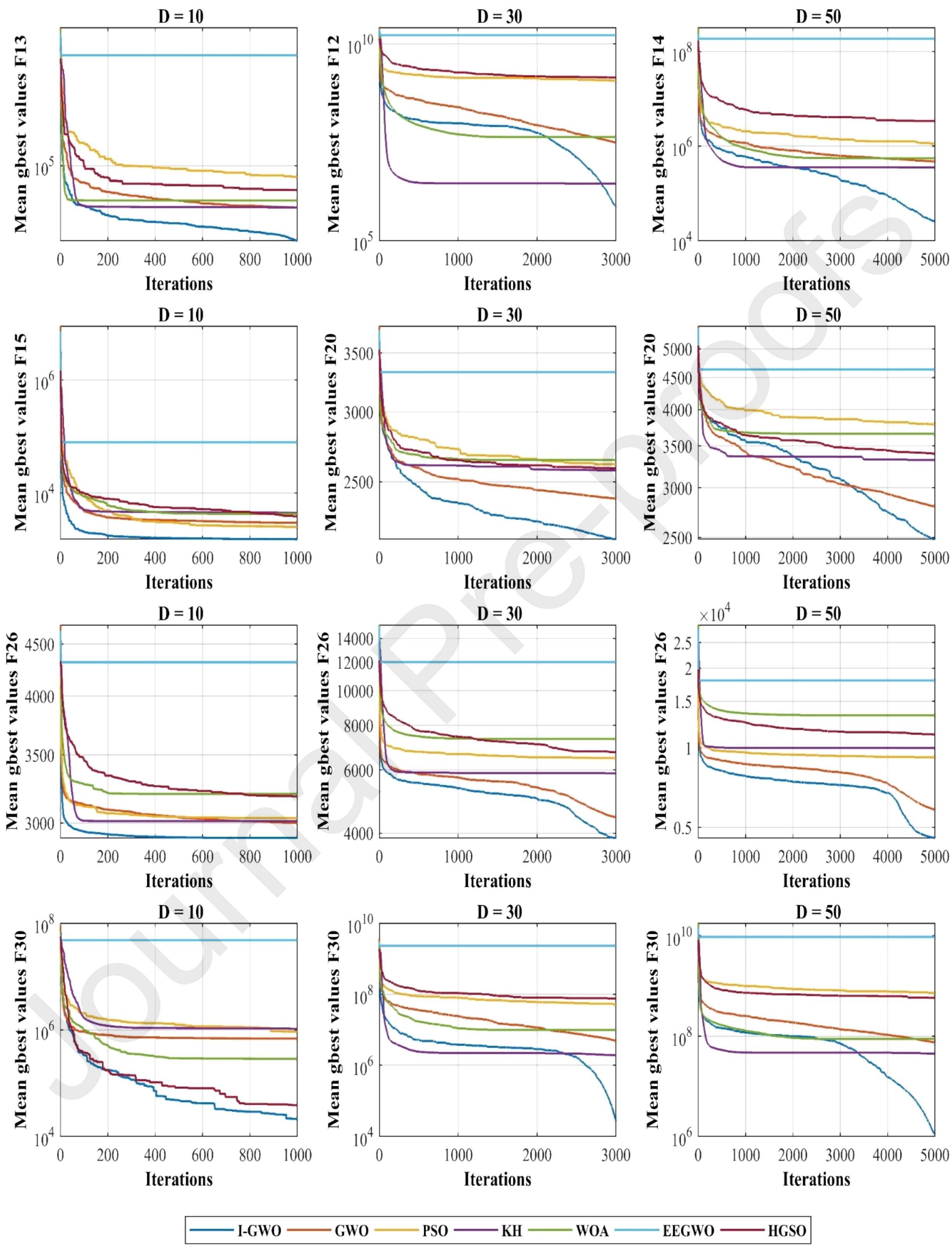

Fig. 6. The convergence curves of hybrid and composition functions with different dimensions.

Table 6 summarizes all performance results of I-GWO and other algorithms shown in Tables 2-5 by a useful metric named overall effectiveness (OE). The OE of each algorithm is computed by Eq. (14), where N is the total number of tests, and L is the total number of losing tests for each algorithm.

$$Overall\ Effectiveness\ (OE)=((N-L)/N)*100 \qquad (14)$$

Table 6 Overall Effectiveness of the I-GWO and other state-of-the-art competitor algorithms.

	PSO (w/t/l)	KH (w/t/l)	GWO (w/t/l)	WOA (w/t/l)	EEGWO (w/t/l)	HGSO (w/t/l)	I-GWO (w/t/l)
D=10	0/0/29	2/0/27	0/0/29	0/0/29	0/0/29	0/0/29	27/0/2
D=30	0/0/29	1/0/28	1/0/28	0/0/29	0/0/29	0/0/29	27/0/2
D=50	0/0/29	0/0/29	1/0/28	0/0/29	0/0/29	0/0/29	28/0/1
Total	0/0/87	3/0/84	2/0/85	0/0/87	0/0/87	0/0/87	82/0/5
OE	0%	3.4%	2.2%	0%	0%	0%	94.2%

5.5 Statistical analysis

Although the obtained results from the experimental evaluation show that the I-GWO algorithm outperforms the comparative algorithms, the rank of the algorithms in these experiments is not determined. Therefore, Friedman and mean absolute error (MAE) tests are conducted to prove the superiority of I-GWO.

5.5.1 Non-parametric Friedman test

In the first statistical test, the Friedman test (F_f) (Derrac et al., 2011) is used for ranking I-GWO and other algorithms based on their obtained fitness using Eq. 15 where k is the number of algorithms, Rj is the mean rank of the j-th algorithm, n is the number of case tests. The test is performed by assuming $\chi 2$ distribution with k-1 degrees of freedom. It first finds the rank of algorithms individually and then calculates the average rank to get the final rank of each algorithm for the considered problem.

$$F_f = \frac{12n}{k(k+1)}\left[\sum_j R_j^2 - \frac{k(k+1)^2}{4}\right] \qquad (15)$$

Inspecting the results in Table 7, it is evident that the I-GWO algorithm significantly differs from other algorithms on the majority of test functions. The overall rank shows that our I-GWO algorithm is better than other algorithms for all dimensions 10, 30, and 50.

Table 7. Overall rank by Friedman test in dimension D = 11, 30, and 50.

Alg.	D	F1	F3	F4	F5	F6	F7	F8	F9	F10	F11	F12	F13	F14	F15	F16	F17
PSO	10	5.40	5.85	5.10	4.00	4.40	6.05	5.65	5.45	4.75	5.25	5.75	5.50	3.65	3.15	3.20	4.95
	30	5.50	4.50	5.00	4.45	3.75	6.70	5.60	4.85	6.00	5.80	5.60	5.45	3.60	6.00	4.55	4.95
	50	5.45	4.40	5.00	5.25	3.70	7.00	5.05	3.95	5.95	5.40	5.00	5.60	4.55	6.00	5.60	6.00
KH	10	**1.00**	4.15	2.55	3.00	2.90	**1.70**	2.60	1.80	3.65	2.95	3.05	3.35	4.10	4.55	5.50	3.00
	30	**1.20**	4.05	1.80	2.75	3.25	2.20	2.80	3.15	2.60	3.40	2.00	2.00	4.15	2.10	3.05	3.05
	50	**1.00**	4.55	**1.10**	2.80	3.30	2.35	2.95	3.00	2.75	4.10	**1.05**	**1.05**	**2.30**	**1.10**	2.80	3.10
GWO	10	3.30	2.55	3.55	1.85	2.10	2.40	2.30	2.80	2.20	2.40	2.55	3.00	3.30	3.35	2.45	2.80
	30	4.00	2.45	3.25	1.90	2.00	1.90	2.00	2.00	**1.45**	2.65	3.35	3.00	2.45	3.30	2.00	2.20
	50	3.55	2.00	3.30	1.65	1.50	**1.80**	**1.45**	**1.57**	1.75	2.50	3.15	3.15	**2.30**	3.10	**1.60**	**1.52**
WOA	10	3.70	2.90	3.25	5.20	5.20	4.70	4.85	5.60	4.00	4.25	3.65	3.80	3.75	4.30	4.10	4.70
	30	3.00	6.60	3.55	4.80	5.50	4.85	4.30	5.25	3.75	3.05	3.60	3.65	4.65	3.65	4.65	4.70
	50	2.00	2.40	2.35	4.10	5.25	4.95	4.10	4.90	4.00	**1.15**	2.75	2.60	3.30	2.75	5.05	4.65
EEGWO	10	7.00	7.00	7.00	7.00	7.00	6.90	6.95	6.90	7.00	7.00	7.00	7.00	6.90	6.95	7.00	6.90
	30	7.00	6.25	7.00	7.00	7.00	6.25	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00
	50	7.00	7.00	7.00	7.00	7.00	6.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00
HGSO	10	5.60	4.55	5.30	5.50	5.25	4.35	4.40	4.05	5.40	5.15	4.90	4.35	5.30	4.70	4.75	4.30
	30	5.50	3.15	6.00	5.75	5.50	4.20	5.10	4.75	4.20	5.10	5.40	5.55	5.15	4.95	5.70	5.10
	50	5.55	5.80	6.00	5.65	5.75	4.05	5.85	5.90	5.00	5.35	6.00	5.40	6.00	5.00	4.25	4.20
I-GWO	10	2.00	**1.00**	**1.25**	**1.45**	**1.15**	1.90	**1.25**	**1.40**	**1.00**	**1.00**	**1.10**	**1.00**	**1.00**	**1.00**	**1.00**	**1.35**
	30	1.80	**1.00**	**1.40**	**1.35**	**1.00**	**1.90**	**1.20**	**1.00**	3.00	**1.00**	**1.05**	**1.35**	**1.00**	**1.00**	**1.05**	**1.00**
	50	3.45	**1.85**	3.25	**1.55**	**1.50**	1.85	1.60	1.68	**1.55**	2.50	3.05	3.20	2.55	3.05	1.70	1.52

Alg.		F													Avg. Rank	Overall Rank
		F18	F19	F20	F21	F22	F23	F24	F25	F26	F27	F28	F29	F30		
PSO	10	4.60	3.35	3.80	4.95	5.60	3.50	5.20	5.55	3.90	3.75	4.05	3.45	5.20	4.66	6
	30	5.15	5.95	4.60	4.65	5.30	3.55	3.15	6.00	3.80	4.45	5.55	4.60	5.15	4.97	6
	50	5.55	6.00	5.65	4.35	6.00	3.10	3.00	5.20	3.20	4.05	5.15	4.80	5.70	5.02	5
KH	10	2.50	3.60	4.75	2.25	**1.50**	3.90	3.10	2.85	2.65	5.00	3.00	4.10	4.40	3.22	3
	30	2.60	2.25	4.10	2.90	2.00	3.65	4.25	2.60	3.45	5.75	2.00	3.40	2.35	2.93	3
	50	2.45	**1.20**	3.65	2.85	3.55	4.10	4.45	**1.25**	4.15	5.80	**1.30**	3.35	**1.40**	2.72	3
GWO	10	3.85	2.75	2.45	4.05	2.45	1.95	3.60	3.05	2.75	2.10	4.85	2.15	4.00	2.86	2
	30	2.70	2.90	2.40	1.95	4.10	2.00	2.00	3.30	2.20	3.10	4.25	2.20	3.10	2.62	2
	50	**2.17**	2.67	1.75	1.63	1.70	1.45	1.43	3.60	1.45	2.42	3.85	1.50	2.88	2.22	2
WOA	10	3.30	5.35	4.35	5.00	3.90	4.55	4.70	4.15	4.75	3.95	4.30	4.95	3.60	4.30	4
	30	4.25	3.95	4.60	4.85	4.85	4.95	4.80	3.10	5.30	4.70	3.40	5.80	3.60	4.40	4
	50	3.65	3.30	4.75	5.00	4.25	5.15	4.75	1.75	5.85	5.15	2.10	4.95	3.15	3.80	4
EEGWO	10	7.00	7.00	7.00	6.80	7.00	7.00	6.75	7.00	7.00	7.00	6.95	7.00	7.00	6.97	7
	30	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	6.95	7
	50	7.00	7.00	6.95	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	6.96	7
HGSO	10	5.50	4.95	4.50	3.35	5.15	5.90	**1.65**	4.35	5.35	5.20	3.50	5.10	2.20	4.64	5
	30	5.00	4.90	4.30	5.50	3.75	5.80	5.75	5.00	5.10	2.00	4.70	4.00	5.80	4.92	5
	50	4.85	5.00	3.70	5.65	3.90	5.65	5.80	5.80	4.80	**1.00**	5.00	4.90	5.30	5.07	6
I-GWO	10	**1.25**	**1.00**	**1.15**	**1.60**	2.40	**1.20**	3.00	**1.05**	**1.60**	**1.00**	**1.35**	**1.25**	**1.60**	**1.36**	**1**
	30	**1.30**	**1.05**	**1.00**	**1.15**	**1.00**	**1.05**	**1.05**	**1.00**	**1.15**	**1.00**	**1.10**	**1.00**	**1.00**	**1.21**	**1**
	50	2.33	2.83	**1.55**	**1.52**	**1.60**	1.55	1.57	3.40	1.55	2.58	3.60	**1.50**	2.58	**2.21**	**1**

5.5.2 Mean absolute error (MAE) test

The statistical mean absolute error (MAE) is a measure to show how far estimates are from the true values. It is computed by Eq. 16, where *NF* is the number of test functions, f_i is the optimization result obtained by i-th function, and f^* is its global optimum.

$$MAE = \frac{1}{NF}\sum_{i=1}^{NF}\left|f_i - f^*\right| \tag{16}$$

Table 8 indicates the mean of absolute errors between the global optimum and the results obtained by each algorithm for all test functions in different dimensions. The results show that the I-GWO algorithm has the smallest MAE, and it is the first rank and superior to other algorithms in all dimensions 10, 30, and 50.

Table 8. Mean absolute error in different dimensions.

Algorithms	D=10		D=30		D=50	
	MAE	Rank	MAE	Rank	MAE	Rank
PSO	7.45E+06	5	4.13E+08	6	1.36E+09	6
KH	3.19E+03	4	2.04E+04	2	7.44E+05	2
GWO	8.02E+02	2	1.51E+06	4	2.92E+07	4
WOA	9.31E+02	3	4.81E+05	3	3.04E+06	3
EEGWO	2.36E+08	7	2.54E+09	7	7.98E+09	7
HGSO	8.28E+06	6	3.96E+08	5	1.25E+09	5
I-GWO	**2.10E+02**	**1**	**5.61E+03**	**1**	**9.66E+04**	**1**

5.6 Impact analyses of DLH

In this experiment, the impact of our DLH search strategy on the performance of the GWO algorithm is analyzed. The results of these analyses are shown in Figs. 7 and 8 on four functions F1, F6, F18, and F26, each selected from different categories of CEC 2018. In I-GWO, first, for each wolf, a candidate solution is generated by the GWO search strategy. Then, by using the new candidate and the current position of the wolf, a neighbor around the X_i is created by Eqs. (10 and 11). After that, by learning from the neighbors, another candidate position is created by the DLH search strategy. Finally, I-GWO selects each produced candidate position that has better fitness for the next iteration. To analyze the impact of the DLH search strategy in GWO, three algorithms GWO, DLH, and GWO+DLH (I-GWO) are developed and compared.

Fig. 7 shows the best fitness of all wolves in each iteration on some selected functions CEC 2018 different dimensions. As the curves shown in this figure, the obtained solutions by DLH are better than GWO for unimodal function F1 and the multimodal function F6, which shows the impact of using DLH in the exploitation and exploration. The DLH search strategy also can find better solutions for the hybrid function F18 and for the composition function F26, which need the balance between exploratory and exploitative searching behaviors. Although the proposed DLH can find the solutions, using both GWO and DLH has more benefits since the solutions obtained by I-GWO are always better than them.

In addition, in this experiment, the percent of improved solutions by DLH and GWO or their improved rate in each iteration for the same functions of the last experiment are shown in Figure 8. All curves of this figure show that the DLH search strategy has more effect on the optimization process, although, in some functions, GWO tends to perform better in the first iterations.

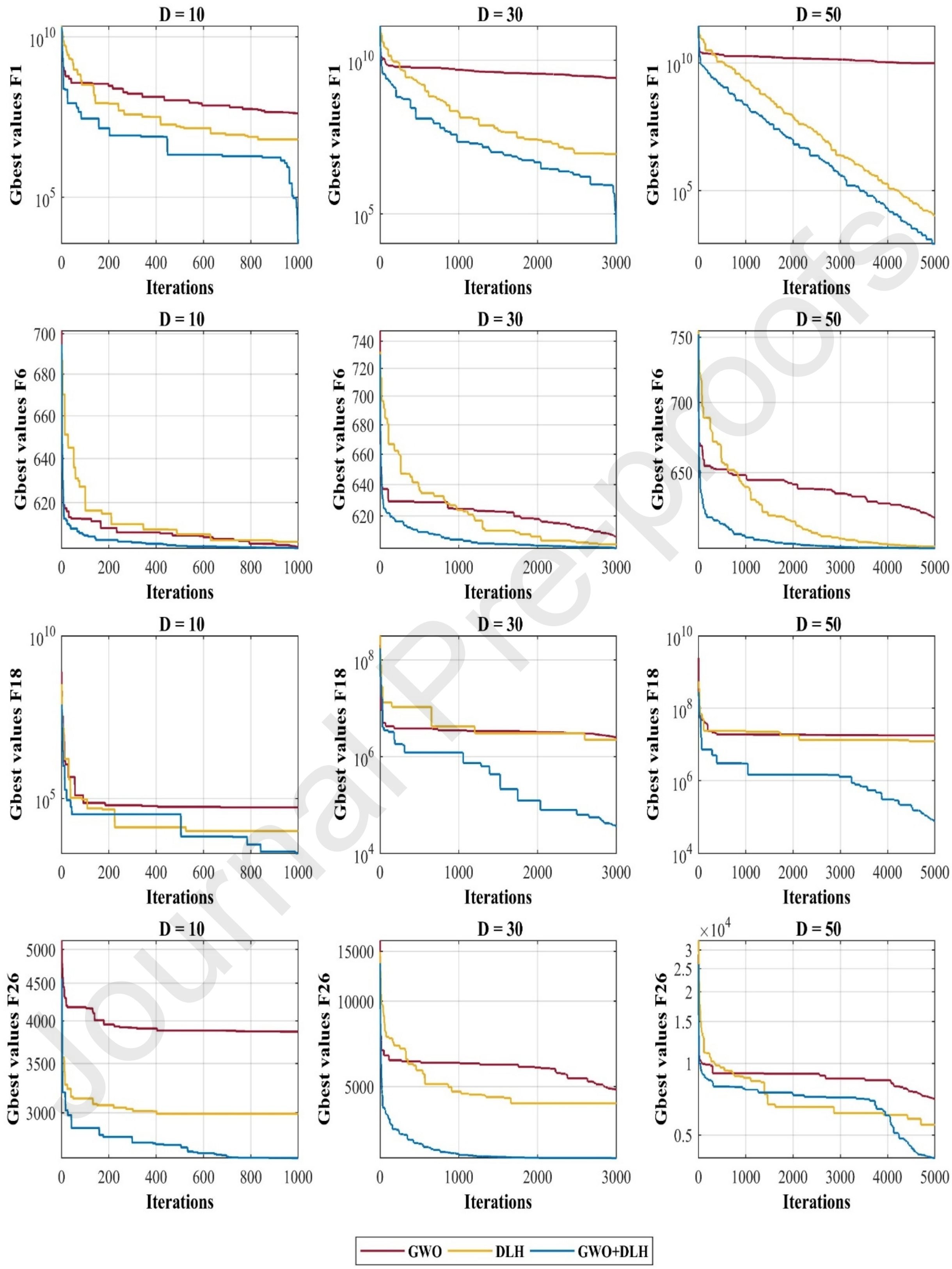

Fig. 7. The best fitness values obtained by GWO, DLH, and GWO+DLH (I-GWO).

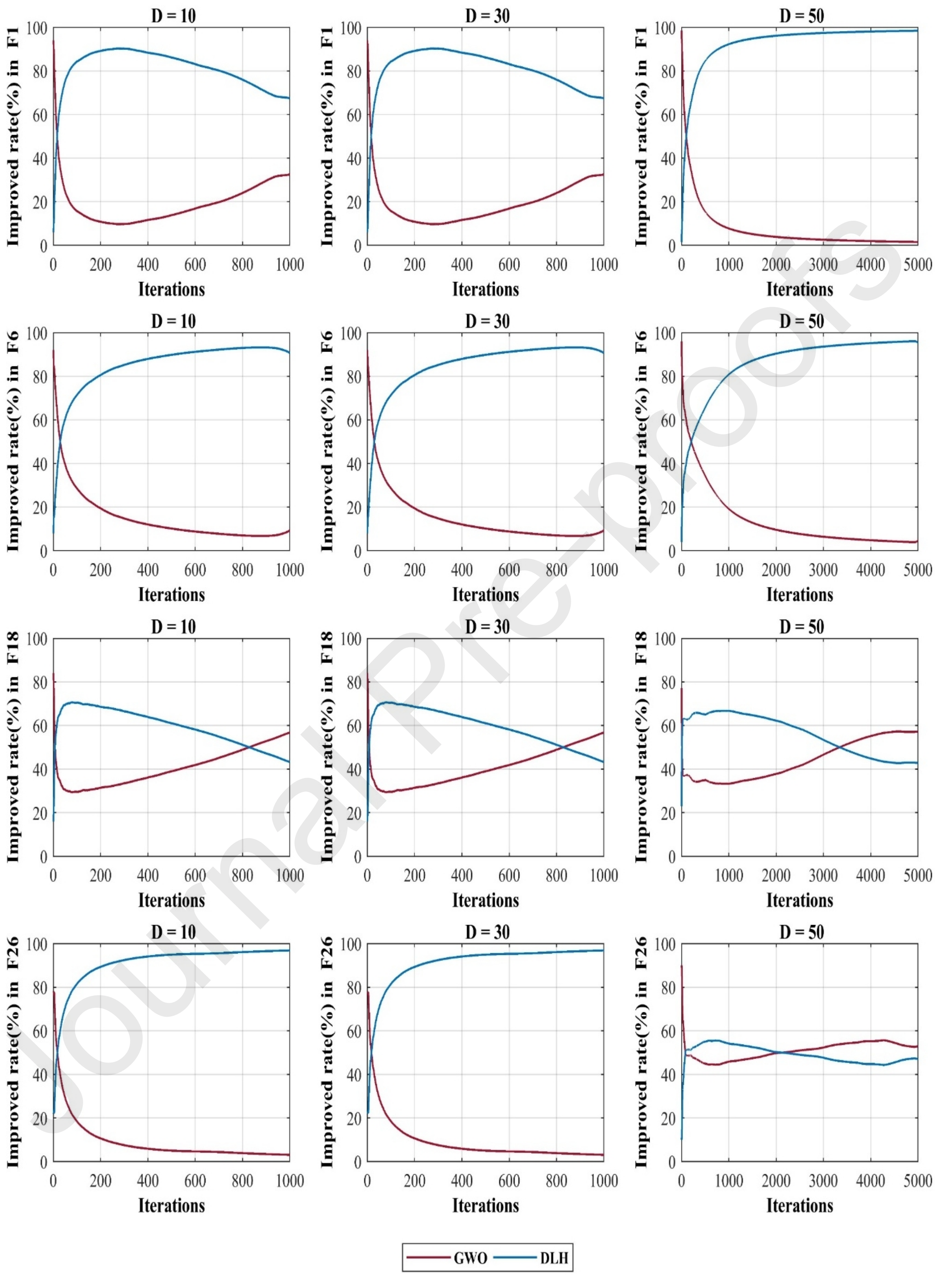

Fig. 8. The rate of improved solutions by GWO and DLH search strategies.

6. Applicability of I-GWO for solving engineering problems

In this section, the applicability of the I-GWO for solving the engineering problems is demonstrated by solving four problems described in the Appendix, including the pressure vessel design, the welded beam design, and the optimal power flow problems for IEEE 30-bus and IEEE 118-bus systems. These problems have several equality and inequality constraints; therefore, I-GWO should handle them during the optimization process. There are several strategies of constraint handling in the literature: rejecting, penalizing, repairing, decoding, and preserving (Talbi, 2009). The death penalty function used by the I-GWO is one of the simplest techniques among the different constraint handling strategies to handle the multi-constraint problems. This function assigns a large fitness value to those solutions that violate any of the constraints to discard the infeasible solutions. The results provided by I-GWO were evaluated and compared with other state-of-the-art algorithms for solving these problems.

In the two first experiments, algorithms were run 10 times, and the swarm size (N) and maximum iterations (Maxiter) were set to 20 and $(D\times10^4)/N$, respectively. Also, in the last two experiments, they set to 20, 50, and 200. The results obtained for the decision variables (DVs) and objective variables are reported in tables 9 to 13, which show that I-GWO, in comparison by other algorithms, provides a design with the best optimal values for these four engineering problems.

Table 9. Results for the pressure vessel problem.

Algorithms	Optimum values for DVs				Optimum Cost
	T_s	T_h	R	L	
PSO	0.883044	0.533053	45.38829	190.0616	7865.2330
KH	0.812418	0.405235	42.08876	177.4774	5975.5430
GWO	0.778709	0.386125	40.34139	199.711	5890.8880
WOA	0.795325	0.396759	40.7552	194.0239	5986.1040
EEGWO	13.09291	6.792196	42.09758	176.6495	6059.8704
HGSO	1.272354	0.624693	65.46899	10	7433.4480
I-GWO	0.779031	0.385501	40.36313	199.4017	**5888.3400**

Table 10. Results for the welded beam problem.

Algorithms	Optimum values for DVs				Optimum Cost
	h	l	t	b	
PSO	0.189561	3.749106	9.31379	0.207391	1.798236
KH	0.150197	5.31408	9.045851	0.205745	1.861801
GWO	0.205409	3.478839	9.035941	0.205774	1.725700
WOA	0.189953	3.99627	8.71047	0.227041	1.871528

EEGWO	0.575855	1.713498	5.719196	0.796379	4.070917
HGSO	0.139093	5.738405	9.313378	0.207628	1.958934
I-GWO	0.20573	3.47049	9.036624	0.20573	**1.724853**

Table 11. Results for the optimal power flow problem using IEEE 30-bus test system.

	Case 1							Case 2						
DVs	PSO	KH	GWO	WOA	EEGWO	HGSO	I-GWO	PSO	KH	GWO	WOA	EEGWO	HGSO	I-GWO
P_{G1} (MW)	184.403	179.069	174.565	178.805	165.303	159.514	176.368	176.010	168.908	145.215	176.996	170.963	163.265	175.880
P_{G2} (MW)	53.294	48.886	48.246	43.572	53.426	38.226	48.847	46.558	46.522	58.991	39.732	36.317	44.769	47.738
P_{G5} (MW)	21.631	21.496	23.502	23.058	19.600	15.868	21.312	21.619	20.864	26.123	23.435	17.774	15.435	24.692
P_{G8} (MW)	12.305	18.035	19.791	20.004	27.992	32.980	21.778	18.394	23.554	30.949	17.597	15.389	21.369	21.003
P_{G11} (MW)	10.505	13.476	13.300	14.994	13.835	18.915	11.140	18.695	16.091	14.851	18.724	21.799	21.097	10.546
P_{G13} (MW)	12.000	12.000	12.970	12.000	12.853	26.126	12.599	12.000	16.993	15.191	16.712	31.234	26.831	13.295
V_{G1} (p.u.)	1.100	1.067	1.084	1.079	1.084	1.079	1.100	1.042	1.035	1.049	1.027	1.066	1.039	1.042
V_{G2} (p.u.)	1.070	1.051	1.066	1.059	1.051	1.069	1.086	1.029	1.013	1.033	1.014	1.049	1.034	1.022
V_{G5} (p.u.)	1.058	1.018	1.032	1.035	0.993	1.062	1.059	0.989	1.020	1.020	1.004	1.011	0.983	1.016
V_{G8} (p.u.)	1.030	1.018	1.035	1.035	1.025	1.048	1.069	0.985	1.001	1.000	1.012	0.977	1.002	1.002
V_{G11} (p.u.)	0.950	1.035	1.052	1.063	0.955	1.014	1.098	1.100	1.030	1.010	1.053	1.015	1.065	1.062
V_{G13} (p.u.)	1.026	1.069	1.040	1.038	1.038	1.003	1.097	0.998	1.040	1.003	1.010	1.013	0.977	0.999
$T_{11(6\text{-}9)}$ (p.u.)	1.100	0.997	0.989	1.019	1.069	1.041	0.967	1.039	0.971	0.997	0.981	0.916	0.995	1.071
$T_{12(6\text{-}10)}$ (p.u.)	0.900	0.988	1.086	0.994	1.061	0.971	1.040	0.902	0.961	0.913	0.944	0.903	1.008	0.901
$T_{15(4\text{-}12)}$ (p.u.)	0.900	1.031	1.000	1.021	0.932	0.975	1.024	0.935	0.987	0.948	0.986	1.062	0.916	0.945
$T_{36(28\text{-}27)}$ (p.u.)	0.999	0.953	0.997	1.015	1.008	1.071	0.996	0.972	0.964	0.958	0.957	0.966	0.967	0.957
Q_{C10} (MVAR)	5.000	2.088	0.483	2.201	2.632	0.361	3.709	3.574	2.080	1.967	0.357	1.930	3.266	2.959
Q_{C12} (MVAR)	0.200	2.723	1.222	3.965	0.099	3.068	3.517	0.000	2.281	1.557	0.584	2.458	0.712	1.796
Q_{C15} (MVAR)	5.000	2.318	0.431	3.562	4.560	0.580	3.563	0.000	1.798	4.142	3.343	2.164	0.431	3.000
Q_{C17} (MVAR)	5.000	3.261	2.215	4.597	4.581	2.570	3.369	2.754	2.483	2.464	4.523	4.959	0.356	2.412
Q_{C20} (MVAR)	0.000	2.365	1.921	5.000	3.855	4.667	4.126	0.000	2.241	2.620	3.869	2.430	2.228	4.756
Q_{C21} (MVAR)	4.078	2.352	3.730	1.875	4.433	4.109	3.520	3.471	2.423	4.097	0.603	2.866	1.933	3.547
Q_{C23} (MVAR)	5.000	2.183	4.163	5.000	3.984	1.519	2.934	4.917	3.826	1.089	2.762	3.322	4.111	2.786
Q_{C24} (MVAR)	0.000	3.006	2.905	5.000	2.116	3.849	4.201	0.182	2.852	4.178	2.537	1.973	0.712	4.169
Q_{C29} (MVAR)	5.000	2.139	0.003	4.735	4.016	4.500	3.743	4.874	3.039	4.387	2.130	3.458	1.861	1.676
Cost ($/h)	805.291	802.027	801.617	801.817	806.621	815.906	**799.340**	805.459	806.285	812.395	808.216	823.047	816.400	**804.640**
Ploss (MW)	10.737	9.562	8.974	9.033	9.610	8.229	8.644	9.876	9.532	7.920	9.796	10.077	9.366	9.753
VD (p.u.)	0.413	0.371	0.387	0.468	0.530	0.534	1.407	0.215	0.162	0.146	0.159	0.295	0.348	0.118

Table 12. Results for the optimal power flow problem using IEEE 118-bus test system for Case 1.

DVs	PSO	KH	GWO	WOA	EEGWO	HGSO	I-GWO	DVs	PSO	KH	GWO	WOA	EEGWO	HGSO	I-GWO	DVs	PSO	KH	GWO	WOA	EEGWO	HGSO	I-GWO
P_{G01}	100.00	63.99	44.15	31.56	68.93	33.80	53.80	P_{G100}	352.00	277.31	78.86	227.12	307.54	182.00	220.11	V_{G74}	1.06	0.99	0.96	1.00	1.04	1.00	0.98
P_{G04}	100.00	43.12	67.03	10.67	59.77	69.81	72.75	P_{G103}	0.00	92.19	79.06	28.51	31.51	14.25	50.65	V_{G76}	1.06	1.03	0.98	1.00	0.99	1.01	0.98
P_{G06}	0.00	83.32	51.47	79.00	99.41	11.67	40.41	P_{G104}	0.00	85.66	32.21	4.08	74.02	44.20	15.99	V_{G77}	1.06	0.97	0.97	1.01	0.96	1.06	0.99
P_{G08}	0.00	21.41	66.11	19.28	35.16	44.95	82.17	P_{G105}	0.00	28.18	19.40	74.60	72.67	54.88	49.07	V_{G80}	1.06	0.95	0.96	1.01	1.02	1.01	0.98
P_{G10}	369.27	50.82	316.49	67.17	250.33	87.33	98.84	P_{G107}	0.00	80.56	2.94	24.64	70.42	32.66	50.93	V_{G85}	0.97	0.97	0.99	1.01	0.95	1.04	1.01
P_{G12}	185.00	54.87	155.30	49.67	18.33	56.23	105.12	P_{G110}	0.00	61.60	6.14	46.70	18.98	51.00	76.58	V_{G87}	0.94	0.98	0.99	1.00	1.01	0.96	1.02
P_{G15}	0.00	16.18	37.85	46.06	39.77	66.12	52.56	P_{G111}	0.00	108.41	73.20	114.10	77.31	74.39	12.07	V_{G89}	0.94	1.04	1.04	1.00	1.01	0.99	1.03
P_{G18}	100.00	73.77	36.07	86.08	56.74	37.62	51.70	P_{G112}	0.00	17.40	58.43	36.74	32.98	49.28	18.38	V_{G90}	0.94	1.05	1.05	1.01	0.94	1.01	1.03
P_{G19}	100.00	24.16	36.81	64.94	23.12	57.31	54.44	P_{G113}	0.00	53.22	45.77	25.44	24.36	63.99	60.67	V_{G91}	0.94	1.03	0.96	1.01	1.01	0.94	0.97
P_{G24}	0.00	83.05	53.87	23.63	52.58	34.87	32.97	P_{G116}	0.00	80.78	40.63	45.99	10.31	34.80	26.52	V_{G92}	0.94	0.99	0.95	1.01	1.00	0.97	1.00
P_{G25}	0.00	256.00	134.84	41.57	243.82	39.89	95.95	V_{G01}	0.94	1.04	1.03	1.01	1.01	1.05	0.97	V_{G99}	0.94	0.99	1.03	1.00	1.01	0.99	0.98
P_{G26}	0.00	296.56	49.17	251.51	230.23	407.30	112.95	V_{G04}	0.94	1.02	1.00	1.01	1.00	0.99	1.02	V_{G100}	0.96	1.01	0.99	1.01	0.98	1.04	1.00
P_{G27}	100.00	54.21	2.38	85.13	51.47	98.76	9.75	V_{G06}	0.94	1.02	1.01	1.01	1.01	0.95	1.02	V_{G103}	0.94	1.01	0.98	1.00	0.96	0.96	1.01
P_{G31}	0.00	39.43	15.93	39.33	20.12	21.86	26.17	V_{G08}	1.06	1.03	1.03	1.01	1.00	1.03	0.99	V_{G104}	0.94	1.00	0.96	1.01	1.01	1.03	0.99
P_{G32}	0.00	70.26	10.12	73.25	5.67	81.59	59.72	V_{G10}	1.06	1.04	1.03	1.00	0.99	0.99	1.06	V_{G105}	0.94	1.00	0.97	1.01	0.97	0.98	1.00
P_{G34}	0.00	87.62	51.18	35.57	52.05	67.99	26.81	V_{G12}	0.94	1.06	1.03	1.01	1.00	0.96	1.00	V_{G107}	0.94	0.98	1.00	1.01	0.99	1.04	1.03
P_{G36}	0.00	36.18	73.59	37.84	55.78	76.27	84.54	V_{G15}	0.94	1.04	1.02	1.01	0.97	0.99	0.98	V_{G110}	1.01	0.97	0.97	1.00	0.95	0.98	1.00
P_{G40}	100.00	40.81	69.97	51.00	64.45	90.25	49.57	V_{G18}	0.94	1.03	1.01	1.01	1.04	1.04	0.98	V_{G111}	1.06	1.02	0.95	1.00	0.94	1.02	0.98
P_{G42}	0.00	26.53	24.82	17.67	5.62	62.22	6.42	V_{G19}	0.94	1.03	1.01	1.01	1.00	1.01	0.97	V_{G112}	1.06	0.99	1.03	1.00	0.98	1.06	1.00
P_{G46}	0.00	37.26	49.94	45.09	76.95	66.59	54.25	V_{G24}	1.06	0.96	0.96	1.01	1.04	0.97	1.01	V_{G113}	0.94	1.01	1.04	1.01	1.02	1.06	0.97
P_{G49}	304.00	88.16	2.40	160.59	22.50	169.95	184.86	V_{G25}	1.06	1.03	0.97	1.00	1.02	0.97	1.04	V_{G116}	1.06	0.97	0.94	1.00	1.03	0.97	0.96
P_{G54}	0.00	29.13	61.37	42.77	110.75	147.00	27.16	V_{G26}	0.94	1.02	0.98	1.01	0.95	1.01	1.03	$T_{(5-8)}$	1.10	1.05	0.94	0.98	1.00	0.93	0.96
P_{G55}	100.00	51.87	31.88	8.77	43.36	20.73	39.44	V_{G27}	0.94	1.00	0.94	1.00	1.02	1.00	1.02	$T_{(25-26)}$	0.90	1.06	1.01	1.03	1.00	1.01	1.02
P_{G56}	100.00	28.13	66.73	32.81	43.50	39.52	87.59	V_{G31}	0.94	1.02	1.02	1.00	1.06	1.01	1.02	$T_{(17-30)}$	0.90	0.97	1.07	0.98	0.96	1.03	0.94
P_{G59}	255.00	42.79	83.42	126.51	76.77	60.03	147.06	V_{G32}	0.94	0.97	0.98	1.01	1.01	0.94	1.00	$T_{(37-38)}$	1.10	0.94	0.95	0.98	1.08	0.99	1.04
P_{G61}	260.00	40.76	139.72	66.77	2.04	153.16	139.41	V_{G34}	0.94	0.99	1.03	1.00	0.97	1.00	0.97	$T_{(59-63)}$	1.10	0.99	0.96	0.98	1.07	0.91	1.05
P_{G62}	0.00	17.78	52.81	6.87	11.96	8.87	45.99	V_{G36}	0.94	0.96	1.04	1.00	0.95	1.01	0.97	$T_{(61-64)}$	1.10	1.05	0.94	1.02	1.03	0.97	0.93
P_{G65}	0.00	298.82	285.63	122.75	404.89	450.32	193.55	V_{G40}	0.94	0.99	1.01	1.00	1.04	0.95	1.02	$T_{(65-66)}$	1.10	1.05	0.97	0.98	0.93	1.08	0.99
P_{G66}	492.00	103.20	304.33	402.91	97.52	258.07	87.04	V_{G42}	0.94	0.99	0.95	1.00	1.05	0.98	1.02	$T_{(68-69)}$	1.10	1.06	0.90	0.98	0.91	1.09	1.01
P_{G70}	0.00	11.69	6.93	74.14	75.64	25.68	71.01	V_{G46}	1.06	0.99	1.01	1.00	1.02	1.01	0.98	$T_{(80-81)}$	0.90	1.03	0.96	0.98	1.05	0.98	1.01
P_{G72}	100.00	4.39	30.62	53.41	72.78	71.73	68.67	V_{G49}	1.01	1.01	0.96	1.00	0.96	0.99	0.99	Q_{C34}	0.00	27.21	22.22	22.64	7.40	5.59	23.05
P_{G73}	0.00	70.09	21.53	3.99	90.98	65.66	77.96	V_{G54}	1.06	0.99	0.95	1.01	1.01	0.95	0.96	Q_{C44}	30.00	11.33	22.69	19.53	19.17	16.08	10.10
P_{G74}	100.00	20.40	15.49	84.20	36.52	9.80	88.44	V_{G55}	1.06	0.98	0.94	1.01	0.97	0.97	0.95	Q_{C45}	30.00	4.04	14.10	10.02	12.28	22.65	20.33
P_{G76}	0.00	77.91	55.91	46.97	92.70	83.56	75.73	V_{G56}	1.06	0.99	0.94	1.00	1.00	0.97	0.96	Q_{C46}	30.00	18.74	21.38	24.87	4.15	11.22	9.40
P_{G77}	0.00	77.57	49.11	31.72	89.91	8.28	46.13	V_{G59}	0.94	1.00	0.96	1.01	0.99	1.02	0.97	Q_{C48}	0.00	19.53	6.06	5.42	7.69	22.43	20.74
P_{G80}	577.00	250.31	180.57	461.22	407.97	235.11	353.08	V_{G61}	0.94	1.02	0.97	1.01	1.03	0.98	1.01	Q_{C74}	0.00	6.51	4.42	16.65	22.77	3.09	4.44
P_{G85}	100.00	30.62	84.22	6.11	99.56	19.42	33.06	V_{G62}	0.94	0.97	0.96	1.00	1.02	0.95	1.01	Q_{C79}	0.00	18.57	12.10	20.33	23.22	24.94	12.36
P_{G87}	0.00	46.05	37.86	8.57	23.99	6.99	16.89	V_{G65}	1.06	1.01	0.96	1.01	0.99	1.05	1.00	Q_{C82}	0.00	23.00	25.36	2.08	17.39	22.58	13.87
P_{G89}	125.44	526.05	604.19	128.16	38.86	155.77	198.50	V_{G66}	0.94	1.02	0.99	1.01	1.00	0.97	1.02	Q_{C83}	0.00	6.42	5.23	20.19	21.00	29.62	14.73
P_{G90}	100.00	18.63	16.63	37.80	12.95	31.81	79.79	V_{G69}	1.06	0.98	0.98	1.00	1.03	1.05	0.99	Q_{C105}	30.00	21.19	15.34	1.89	19.63	18.28	17.74
P_{G91}	0.00	52.18	7.72	31.91	3.54	63.00	43.75	V_{G70}	1.06	1.00	0.97	1.01	0.98	1.02	0.99	Q_{C107}	30.00	6.31	7.71	25.65	24.61	20.12	11.38
P_{G92}	0.00	56.27	13.38	67.32	35.66	55.81	33.10	V_{G72}	1.06	1.01	1.05	1.01	0.98	1.06	1.04	Q_{C110}	0.00	6.80	11.72	13.65	9.43	2.34	24.33
P_{G99}	0.00	37.37	34.86	74.69	90.27	43.12	31.78	V_{G73}	1.06	1.00	0.98	1.01	0.96	0.95	0.96								

Final results	PSO	KH	GWO	WOA	EEGWO	HGSO	I-GWO
Cost ($/h)	151751.61	155696.34	145902.97	144856.49	152012.53	150221.08	142980.33
Ploss (MW)	114.431	188.555	135.040	76.802	126.458	104.136	61.019
VD (p.u.)	2.953	1.579	2.217	0.405	1.353	1.460	1.156

Table 13. Results for the optimal power flow problem using IEEE 118-bus test system for Case 2.

DVs	PSO	KH	GWO	WOA	EEGWO	HGSO	I-GWO	DVs	PSO	KH	GWO	WOA	EEGWO	HGSO	I-GWO	DVs	PSO	KH	GWO	WOA	EEGWO	HGSO	I-GWO
P_{G01}	0.00	63.99	35.51	9.88	88.95	33.71	56.02	P_{G100}	0.00	277.31	222.78	192.35	238.99	67.20	325.39	V_{G74}	1.06	0.99	0.99	1.02	1.06	0.98	0.97
P_{G04}	0.00	43.12	62.67	39.89	75.08	24.00	29.66	P_{G103}	140.00	92.19	93.59	55.81	71.89	34.11	30.91	V_{G76}	1.06	1.03	0.96	1.01	1.01	0.96	0.97
P_{G06}	100.00	83.32	39.58	75.85	2.45	79.48	65.70	P_{G104}	100.00	85.66	66.16	56.69	13.40	65.25	70.10	V_{G77}	1.04	0.97	1.00	1.01	1.02	1.06	0.99
P_{G08}	77.21	21.41	58.37	18.35	37.26	45.88	24.13	P_{G105}	0.00	28.18	48.48	43.03	15.58	80.45	42.15	V_{G80}	1.06	0.95	0.99	1.02	1.03	0.98	1.01
P_{G10}	550.00	50.82	141.56	414.95	78.15	259.10	323.66	P_{G107}	100.00	80.56	29.71	51.50	42.48	55.86	29.21	V_{G85}	1.06	0.97	1.04	1.02	1.01	1.04	1.01
P_{G12}	185.00	54.87	40.45	131.16	129.74	181.86	73.31	P_{G110}	0.00	61.60	34.74	59.02	26.38	61.12	49.18	V_{G87}	1.06	0.98	1.04	1.02	1.03	0.99	1.04
P_{G15}	0.00	16.18	28.94	26.00	28.82	17.88	39.14	P_{G111}	136.00	108.41	65.47	35.79	117.75	9.16	37.30	V_{G89}	1.06	1.04	1.02	1.02	1.00	0.99	1.03
P_{G18}	0.00	73.77	91.46	4.57	92.27	38.68	46.65	P_{G112}	0.00	17.40	37.38	65.49	58.76	83.57	52.80	V_{G90}	0.94	1.05	0.94	1.01	1.01	1.00	0.97
P_{G19}	0.00	24.16	44.95	39.39	83.91	61.35	54.06	P_{G113}	0.00	53.22	76.86	75.50	70.66	39.93	47.65	V_{G91}	0.94	1.03	0.96	1.01	0.97	1.03	0.99
P_{G24}	100.00	83.05	56.92	54.42	25.52	35.92	38.17	P_{G116}	0.00	80.78	46.90	9.97	27.09	15.67	14.51	V_{G92}	0.94	0.99	1.00	1.01	1.00	0.97	0.99
P_{G25}	320.00	256.00	247.63	192.25	318.96	17.59	62.45	V_{G01}	0.94	1.04	0.99	1.01	0.95	1.01	1.03	V_{G99}	0.94	0.99	1.01	1.01	0.95	1.02	1.03
P_{G26}	0.00	296.56	24.02	161.28	363.90	85.07	301.51	V_{G04}	0.94	1.02	1.00	1.02	0.95	1.00	1.03	V_{G100}	0.94	1.01	1.03	1.02	0.99	0.95	1.01
P_{G27}	100.00	54.21	25.06	59.16	8.20	84.22	32.46	V_{G06}	0.94	1.02	1.00	1.01	0.97	1.00	1.02	V_{G103}	0.94	1.01	1.02	1.02	0.95	0.97	1.01
P_{G31}	0.00	39.43	48.92	28.42	80.57	32.89	36.04	V_{G08}	1.06	1.03	0.96	1.01	1.01	0.97	1.00	V_{G104}	0.94	1.00	1.02	1.02	1.04	1.02	0.99
P_{G32}	0.00	70.26	33.75	14.57	60.42	94.30	34.61	V_{G10}	1.06	1.04	1.02	1.02	1.02	1.05	0.98	V_{G105}	0.94	1.00	1.00	1.02	1.02	1.03	0.98
P_{G34}	25.65	87.62	80.68	56.63	58.53	65.55	31.33	V_{G12}	0.94	1.06	1.00	1.02	1.03	1.04	1.02	V_{G107}	0.96	0.98	0.97	1.02	1.06	1.06	0.95
P_{G36}	100.00	36.18	50.67	21.07	65.30	62.25	50.69	V_{G15}	0.94	1.04	0.97	1.01	1.00	1.05	0.97	V_{G110}	0.98	0.97	1.02	1.01	0.98	0.95	1.00
P_{G40}	0.00	40.81	29.47	11.29	30.40	17.50	51.63	V_{G18}	0.94	1.03	0.96	1.02	1.05	0.96	0.98	V_{G111}	0.94	1.02	1.03	1.02	1.01	1.01	1.01
P_{G42}	0.00	26.53	18.09	23.77	73.97	86.67	46.53	V_{G19}	0.94	1.03	0.97	1.02	1.01	1.03	0.97	V_{G112}	1.06	0.99	1.02	1.01	1.03	0.98	1.01
P_{G46}	0.00	37.26	48.03	5.08	8.17	95.45	33.69	V_{G24}	1.06	0.96	0.98	1.02	0.97	1.05	1.00	V_{G113}	0.94	1.01	0.99	1.01	1.05	0.96	0.99
P_{G49}	0.00	88.16	91.60	104.43	57.17	220.74	101.90	V_{G25}	0.94	1.03	1.02	1.02	1.04	1.02	1.00	V_{G116}	1.06	0.97	0.99	1.01	1.01	1.02	0.98
P_{G54}	0.00	29.13	54.37	106.65	20.40	135.79	54.92	V_{G26}	0.94	1.02	0.96	1.01	0.97	0.95	0.98	$T_{(5-8)}$	1.10	1.05	0.90	1.01	1.00	0.99	1.00
P_{G55}	100.00	51.87	48.71	56.25	24.59	80.68	43.44	V_{G27}	1.06	1.00	1.00	1.01	1.03	0.98	0.99	$T_{(25-26)}$	1.10	1.06	0.97	0.99	0.92	0.92	1.00
P_{G56}	0.00	28.13	35.79	71.08	83.46	80.04	36.87	V_{G31}	1.06	1.02	0.98	1.02	0.99	1.03	1.01	$T_{(17-30)}$	1.10	0.97	1.01	0.99	0.91	0.92	1.07
P_{G59}	0.00	42.79	129.98	154.32	188.87	18.37	77.93	V_{G32}	1.06	0.97	0.98	1.01	1.04	0.99	0.98	$T_{(37-38)}$	1.10	0.94	1.08	1.00	1.00	0.98	1.01
P_{G61}	260.00	40.76	136.61	71.06	92.62	113.83	102.17	V_{G34}	0.94	0.99	0.97	1.02	0.98	0.96	0.99	$T_{(59-63)}$	0.90	0.99	0.91	0.98	1.00	0.96	0.96
P_{G62}	0.00	17.78	67.82	40.42	47.98	59.34	51.85	V_{G36}	0.94	0.96	0.96	1.02	1.01	1.00	0.99	$T_{(61-64)}$	0.97	1.05	0.97	0.98	0.94	0.99	0.99
P_{G65}	491.00	298.82	37.63	29.90	228.08	74.88	171.03	V_{G40}	0.94	0.99	1.06	1.02	1.05	1.01	0.99	$T_{(65-66)}$	1.10	1.05	1.00	0.99	1.02	0.91	1.04
P_{G66}	0.00	103.20	463.61	101.59	477.49	177.34	169.53	V_{G42}	0.95	0.99	0.94	1.02	1.05	1.00	1.03	$T_{(68-69)}$	0.90	1.06	1.01	1.00	0.93	1.02	0.99
P_{G70}	0.00	11.69	53.88	4.07	24.23	40.24	82.47	V_{G46}	1.06	0.99	1.02	1.02	0.95	0.97	1.01	$T_{(80-81)}$	0.90	1.03	1.09	1.00	1.08	0.95	0.98
P_{G72}	0.00	4.39	52.27	32.30	63.39	19.82	51.65	V_{G49}	0.98	1.01	1.02	1.01	1.06	1.00	1.00	Q_{C34}	0.00	27.21	20.72	19.93	21.26	10.40	10.19
P_{G73}	0.00	70.09	69.59	43.50	21.95	49.63	52.50	V_{G54}	0.94	0.99	1.05	1.02	0.99	1.01	1.01	Q_{C44}	30.00	11.33	16.63	0.88	7.20	7.68	17.12
P_{G74}	0.00	20.40	40.20	65.04	11.95	8.87	37.28	V_{G55}	0.94	0.98	1.03	1.02	0.95	1.00	1.00	Q_{C45}	30.00	4.04	7.37	20.27	10.68	16.19	10.65
P_{G76}	100.00	77.91	52.41	41.47	98.84	49.84	71.33	V_{G56}	0.94	0.99	1.04	1.02	1.00	1.00	1.00	Q_{C46}	30.00	18.74	19.40	9.81	28.03	0.69	6.21
P_{G77}	0.00	77.57	81.11	17.31	74.94	27.88	62.46	V_{G59}	1.06	1.00	1.01	1.02	0.95	0.94	0.99	Q_{C48}	30.00	19.53	20.39	13.37	6.60	12.00	13.22
P_{G80}	0.00	250.31	273.42	413.55	68.23	342.76	207.35	V_{G61}	1.06	1.02	1.03	1.02	0.98	1.06	1.01	Q_{C74}	30.00	6.51	8.60	21.70	21.34	16.09	18.30
P_{G85}	0.00	30.62	53.24	8.08	13.04	90.00	71.97	V_{G62}	1.06	0.97	1.02	1.02	0.96	1.03	0.99	Q_{C79}	30.00	18.57	22.36	16.34	16.44	16.86	15.82
P_{G87}	0.00	46.05	46.45	16.21	11.74	32.17	21.52	V_{G65}	1.06	1.01	1.01	1.01	1.04	0.95	1.01	Q_{C82}	30.00	23.00	12.81	9.95	6.63	16.99	19.01
P_{G89}	707.00	526.05	233.21	256.32	110.33	259.67	187.17	V_{G66}	1.03	1.02	1.01	1.01	1.04	0.95	1.00	Q_{C83}	0.00	6.42	15.45	19.63	3.97	2.43	22.67
P_{G90}	0.00	18.63	25.35	6.07	11.62	47.58	7.57	V_{G69}	1.06	0.98	1.04	1.02	1.03	1.06	1.02	Q_{C105}	0.00	21.19	12.06	14.78	17.60	6.29	9.39
P_{G91}	26.40	52.18	29.69	80.61	33.05	63.19	28.60	V_{G70}	1.06	1.00	1.03	1.02	1.05	1.06	1.01	Q_{C107}	30.00	6.31	9.60	12.06	6.82	27.79	9.60
P_{G92}	100.00	56.27	59.29	67.63	52.44	46.62	50.13	V_{G72}	0.94	1.01	0.96	1.01	1.04	1.06	1.01	Q_{C110}	0.00	6.80	26.90	9.35	0.46	28.66	16.91
P_{G99}	100.00	37.37	81.55	15.33	24.54	86.46	61.62	V_{G73}	1.06	1.00	1.05	1.02	0.97	1.01	1.05								

Final results	PSO	KH	GWO	WOA	EEGWO	HGSO	I-GWO
Cost ($/h)	163613.92	155696.40	153293.89	145078.86	159448.68	154139.58	142115.60
Ploss (MW)	242.265	188.555	88.693	77.031	156.762	96.002	79.627
VD (p.u.)	3.059	1.579	1.082	0.668	1.057	1.360	0.880

7. Discussion

In this section, the main reasons for the superiority of the I-GWO algorithm over the comparative algorithms are discussed. The results reported in Table 2 and the convergence curves shown in Fig. 5 certify that the I-GWO algorithm is competitive for unimodal problems, especially by increasing the dimensions, and it also has a faster convergence than other algorithms. This is mostly because the I-GWO uses the DLH search strategy based on the positions selected from the neighborhood with the radius defined in Eq. (10) which enhances the exploitation.

The results reported in Table 3 and the curves shown in Fig. 5 indicate that the I-GWO algorithm is competitive with other algorithms for solving multimodal benchmark functions. The main reason for this sufficiency of the proposed algorithm in the exploration and convergence is the neighbors' dimensional learning. Using this learning provides wolves to avoid local optima, which results in exploring the search space extensively. Moreover, the neighborhood structure used in I-GWO is defined by the principle that helps the diversification and intensification in every stage of the optimization. These are satisfied by considering the distance in which the larger distance, the diversity of visited wolves is more. By contrast, the smaller the distance, the number of its neighbors is less.

The convergence curves plotted in Fig. 6 and the results tabulated in Tables 4 and 5 demonstrate the superiority of I-GWO on the majority of hybrid and composition functions. Since these functions including, shifted and rotated unimodal and multimodal functions; therefore, the results prove that I-GWO can appropriately balance exploration and exploitation. Moreover, it maintains the diversity to handle difficulties in these kinds of complex functions. The main reason is having benefits of using both of the GWO and DLH search strategies, which are complementary to enhance the balance between the exploration and exploitation and the local optima avoidance.

8. Conclusion and Future Work

The leadership hierarchy and group hunting mechanism of grey wolves in nature was the inspiration of the GWO algorithm. Because of considering the only three best wolves of the population in the movement strategy, the GWO mostly suffers from lack of the population diversity, imbalance between the exploitation and exploration, and the premature convergence (Heidari et al., 2017; Long et al., 2018; Lu et al., 2018; Tu et al., 2019a). To handle these shortcomings, we proposed an improved version of GWO named improved grey wolf optimizer (I-GWO).

In the proposed I-GWO algorithm, the movement strategy is developed by introducing the dimension learning-based hunting (DLH) search strategy inspired by the individual hunting of the grey wolves. Then, the movement strategy of the I-GWO selects the candidate either from the GWO or the DLH search strategies based on the quality of their new positions. The cooperation among these two search strategies improves the global and local search ability of the proposed algorithm. From the experimental results and the just-mentioned discussions, the following conclusions can be drawn:

- Using the proposed dimension learning-based hunting (DLH) search strategy enhances exploration and exploitation.
- Using the GWO and introduced DLH search strategies together maintains the diversity, enhances the balance between the local and global search strategies, and escapes from the local optima.
- The obtained results from different experiments and statistical tests prove that I-GWO has better performance than the comparative algorithms for benchmark functions with different characteristics.
- The I-GWO algorithm has the potential to solve engineering design problems and optimal power flow problem.

I-GWO is a single-objective algorithm developed for continuous problems; thus, the binary and multi-objective versions of this algorithm can be implemented. Moreover, the proposed algorithm can be modified to solve large scale unconstrained global optimization problems. The proposed algorithm can be adapted for solving other real-world and large-scale optimization problems.

Appendix

- **Pressure vessel design problem**

The main objective of the pressure vessel design problem (Kannan et al., 1994) shown in Fig. A.1is to minimize the cost of material, forming, and welding of a vessel. Four variables of this problem are the thickness of shell (T_s) and head (T_h), inner radius (R), and cylindrical section length without considering the head (L). The mathematical formulation and four constraint functions of this problem are given in Eq. (A.1).

Consider $\vec{x} = [x_1 x_2 x_3 x_4] = [T_s\, T_h\, R\, L]$ (A.1)

Minimize $f(\vec{x}) = 0.6224 x_1 x_3 x_4 + 1.7781 x_2 x_3^2 + 3.1661 x_1^2 x_4 + 19.84 x_1^2 x_3$

Subject to $g_1(\vec{x}) = -x_1 + 0.0193x_3 \leq 0,$

$g_2(\vec{x}) = -x_2 + 0.00954x_3 \leq 0,$

$g_3(\vec{x}) = -\pi x_3^2 x_4 - \frac{4}{3}\pi x_3^3 + 1.296.000 \leq 0,$

$g_4(\vec{x}) = x_4 - 240 \leq 0$

where $0 \leq x_i \leq 100.\ i = 1.\ 2$

$10 \leq x_i \leq 200.\ i = 3.4$

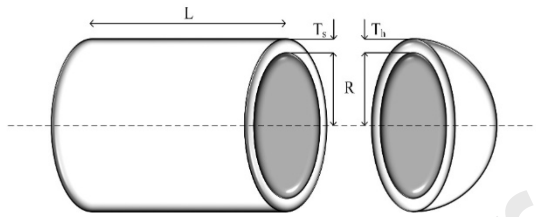

Fig. A.1. Design of pressure vessel problem

- **Welded beam design problem**

The objective of this design problem is to obtain a minimum fabrication cost for designing a welded beam (Coello, 2000). In Fig. A.2, there are four design variables to be optimized: the weld thickness (*h*), the attached part of the bar length (*l*), the bar height (*t*), and the bar thickness (*b*). Also, by applying the load on top of the bar, seven constraints should not be violated. These constraints are shear stress (τ), bending stress in the beam (σ), end deflection of the beam (δ), and buckling load on the bar (P_b). The formulation of this problem is computed by Eq. (A.2).

Consider $\vec{x} = [x_1 x_2 x_3 x_4] = [h\ l\ t\ b]$ (A.2)

Minimize $f(\vec{x}) = 1.10471x_1^2 x_2 + 0.04811x_3 x_4 * (14.0 + x_2)$

Subject to $g_1(\vec{x}) = \tau(\vec{x}) - \tau_{max} \leq 0,$

$g_2(\vec{x}) = \sigma(\vec{x}) - \sigma_{max} \leq 0,$

$g_3(\vec{x}) = \delta(\vec{x}) - \delta_{max} \leq 0,$

$g_4(\vec{x}) = x_1 - x_4 \leq 0$

$g_5(\vec{x}) = \mathrm{P} - P_c(\vec{x}) \leq 0$

$g_6(\vec{x}) = 0.125 - x_1 \leq 0$

$g_7(\vec{x}) = 1.10471x_1^2 + 0.04811x_3 x_4 * (14.0 + x_2) - 0.5 \leq 0$

where $0.1 \leq x_i \leq 2.\ i = 1.\ 2$

$0.1 \leq x_i \leq 10.\ i = 3.4$

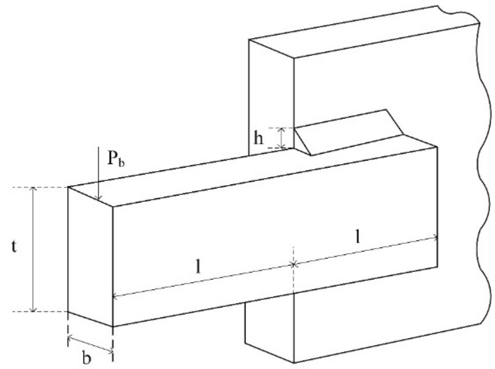

Fig. A.2. Design of welded beam problem

- **Optimal power flow problem for IEEE 30-bus system**

The IEEE 30-bus test system shown in Fig. A.3 (Radosavljević et al., 2015) consists of six generators, four transformers, and nine shunt VAR compensation buses. The lower and upper bounds of the transformer tap are set to 0.9 and 1.1 p.u. The minimum and maximum values of the shunt VAR compensations are 0.0 and 0.05 p.u. The lower and upper limit values of the voltages for all generator buses are set to be 0.95 and 1.1 p.u.

Case 1: Minimization of the fuel cost

In this case, the objective function f_1 signifies the minimization of fuel cost for all generators and is calculated by Eq. (A.3).

$$f_1 = \sum_{i=1}^{NG} (a_i + b_i \, P_{Gi} + c_i \, P_{Gi}^2) \tag{A.3}$$

Where a_i, b_i and c_i are the cost coefficient of the i-th generator. For P_{Gi} (in MW), a_i, b_i, and c_i are considered in \$/hr, \$/MWh, and S/MW^{2}h.

Case 2: Voltage profile improvement

The objective function f_2 is considered to minimize the fuel cost and voltage deviations and calculated by Eq. (A.4).

$$f_2 = \sum_{i=1}^{NG} (a_i + b_i \, P_{Gi} + c_i \, P_{Gi}^2) + W_v \sum_{i=1}^{NL} |V_i - 1.0| \tag{A.4}$$

Where the value of weighting factor W_v is set to 200.

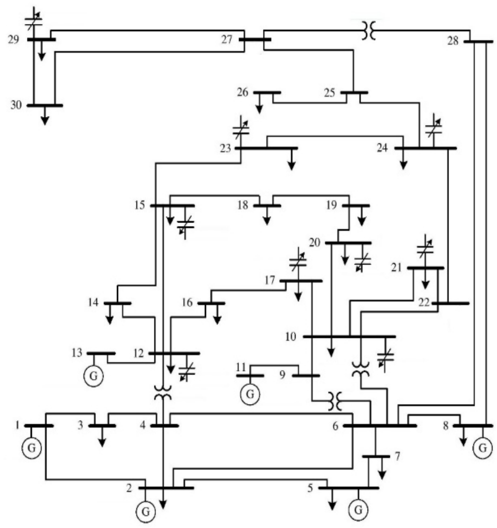

Fig. A.3. IEEE 30-bus test system single-line diagram

- **Optimal power flow problem for IEEE 118-bus system**

The IEEE 118-bus test system is shown in Fig. A.4 (Radosavljević et al., 2015). It may be seen that this problem has 54 generators, 186 branches, nine transformers, two reactors, and 12 capacitors. In this problem, there are 29 control (decision) variables: 54 generator, nine settings for transformers, and 12 shunt capacitor reactive power injections. The voltage limits of all buses are between 0.94 and 1.06 p.u. The transformer tap settings are considered within the interval of 0.90–1.10 p.u. The available reactive powers of shunt capacitors are within the range 0–30 MVAR. To optimize this problem, two optimization cases are considered: to minimize the fuel cost of all generators and to minimize the voltage profile improvement. The objectives are as in Case 1 and Case 2 for the IEEE 30-bus test system.

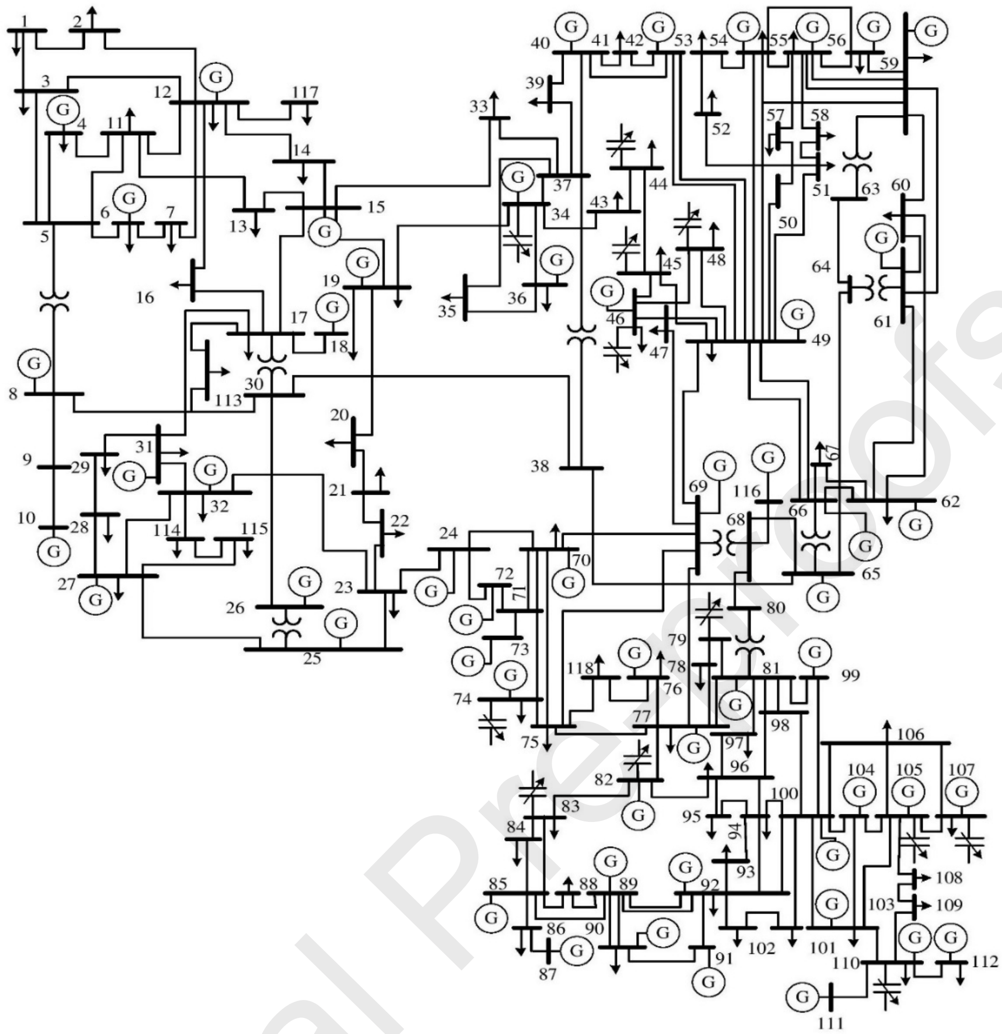

Fig. A.4. IEEE 118-bus test system single-line diagram

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Title: An Improved Grey Wolf Optimizer for Solving Engineering Problems

Mohammad H. Nadimi-Shahraki[*]: Conceptualization, Methodology, Software, Validation, Formal analysis, Investigation, Writing - Original Draft, Writing - Review & Editing, Supervision and Project administration.

Shokooh Taghian: Conceptualization, Methodology, Software, Validation, Formal analysis, Investigation, Writing - Original Draft and Writing - Review & Editing.

Seyedali Mirjalili: Validation, Formal analysis, Writing - Review & Editing and Supervision.

- Proposing an improved Grey Wolf Optimizer (I-GWO) for solving engineering problems
- Introducing a new search strategy named dimension learning-based hunting (DLH)
- DLH is to enhance balance between local and global search and maintain diversity
- Performance of I-GWO is evaluated on the CEC2018 and three engineering problems
- I-GWO algorithm is very competitive and superior to the compared algorithms